Transfer Functions

A. Relate ___single___ input to ___single___ output.

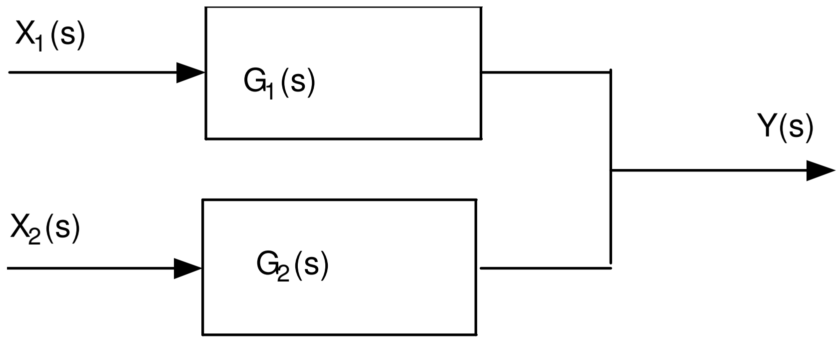
B. Represent an ___algebraic___ relationship in s domain.

C. Can be ___combined___ to give the total system behavior.

D. Convenient to use with ___block___ diagrams.

E. Require initial conditions to be ___0___. (use ___deviation___ variables)

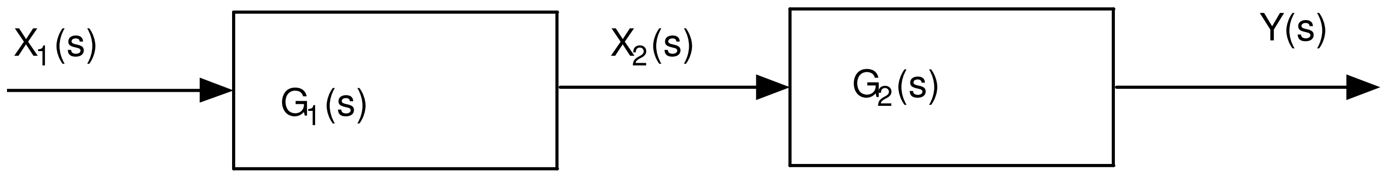
F. Additive property (parallel process)



$$Y(s) = X_1(s)\,G_1(s) + X_2(s)\,G_2(s)$$

Transfer Functions (cont.)

G. Multiplicative property (series process)



$Y(s) = X_2(s)\,G_2(s)$
$X_2(s) = X_1(s)\,G_1(s)$
So $Y(s) = X_1(s)G_1(s)\,G_2(s)$

H. **K** is the limit of G(s) as s approaches 0 (for a unit step change) when the **limit** exists (see pg. 84).

I. Find the gain for the following transfer functions:

$$G_1(s) = \frac{1}{\tau s + 1} \quad = 1$$

$$G_1(s) = \frac{a + bs}{\tau s + 1} \quad = a$$

$$G_2(s) = \frac{8 + 2s}{(s+3)(s+2)} = \frac{8}{(3)(2)} = \frac{4}{3}$$

Group Activity

1. Find $C'(s)/C_i'(s)$

$$\frac{dC'}{dt} = \frac{q}{V}C_i' - \frac{q}{V}C'$$

$$sC'(s) = \frac{q}{V}C_i'(s) - \frac{q}{V}C'(s)$$

$$\left(s + \frac{q}{V}\right)C'(s) = \frac{q}{V}C_i'(s)$$

$$\left(\frac{V}{q}s + 1\right)C'(s) = C_i'(s)$$

$$\frac{C'(s)}{C_i'(s)} = \frac{1}{\left(\frac{V}{q}s + 1\right)}$$

$$\tau_p = \frac{V}{q}$$

$$K_p = 1$$

Group Activity

2. Find $C'(s)/C_i'(s)$

$$\frac{dC'}{dt} = \frac{q}{V}C_i' - \left(\frac{q}{V} + 2k_2\overline{C}\right)C'$$

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$$sC'(s) = \frac{q}{V}C_i'(s) - \frac{1}{\beta}C'(s)$$

$$(\beta s + 1)C'(s) = \beta\frac{q}{V}C_i'(s)$$

$$\frac{C'(s)}{C_i'(s)} = \frac{\beta\frac{q}{V}}{(\beta s + 1)}$$

$$\tau_p = \beta$$

$$K_p = \beta\frac{q}{V}$$

Group Activity

3. Find $T'(s)/T_i'(s)$ and $T'(s)/Q'(s)$

$$\frac{dT'}{dt} = \frac{q}{V}(T_i' - T') + \frac{Q'}{\rho V C_p}$$

$$\beta = \frac{q}{V}$$

$$\alpha = \frac{1}{\rho V C_p}$$

$$\frac{dT'}{dt} = \beta(T_i' - T') + \alpha Q'$$

$$sT'(s) = \beta T_i'(s) - \beta T'(s) + \alpha Q'(s)$$

$$(s + \beta)T'(s) = \beta T_i'(s) + \alpha Q'(s)$$

$$\left(\frac{1}{\beta}s + 1\right)T'(s) = T_i'(s) + \frac{\alpha}{\beta}Q'(s)$$

$$\frac{T'(s)}{T_i'(s)} = \frac{1}{\left(\frac{1}{\beta}s + 1\right)}$$

$$\frac{T'(s)}{Q'(s)} = \frac{\frac{\alpha}{\beta}}{\left(\frac{1}{\beta}s + 1\right)}$$