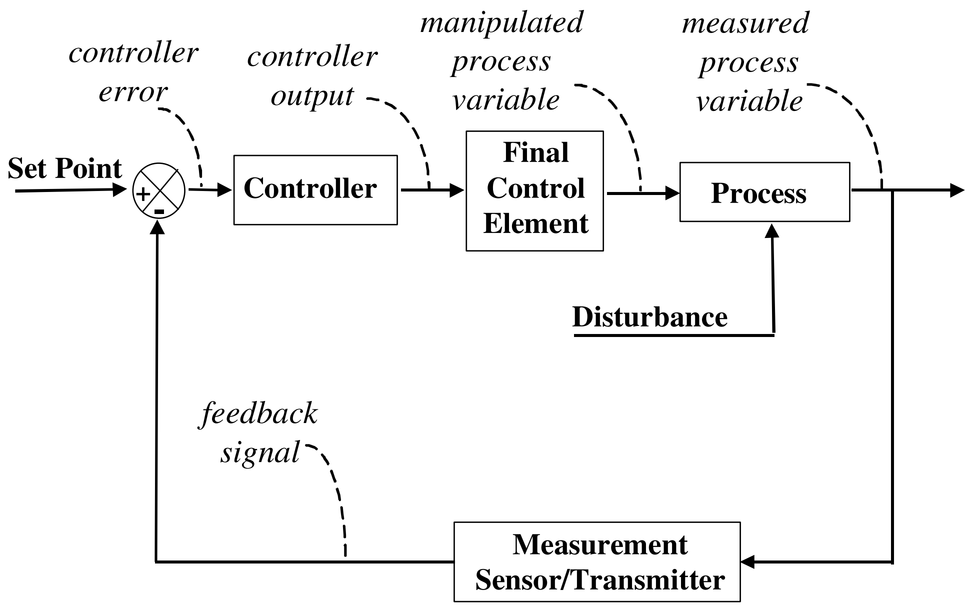
Lecture 29 – Closed Loop Control Design with Overall Transfer Functions

Block Diagram Overview



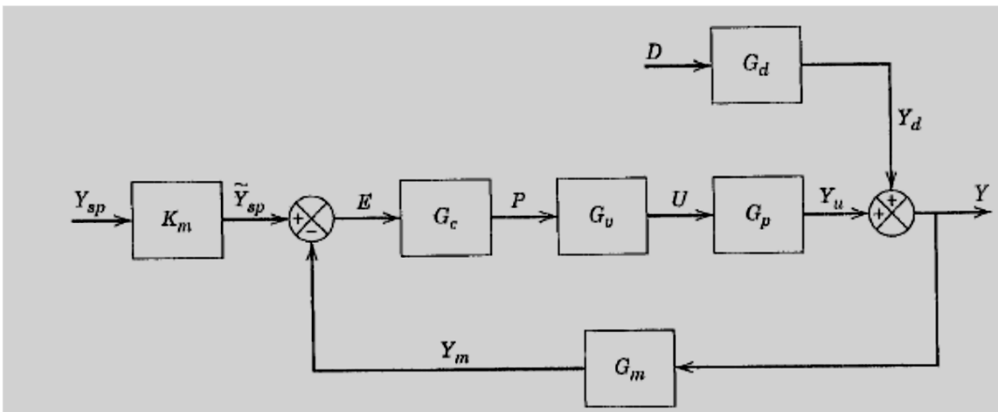


Figure 11.8 Standard block diagram of a feedback control system.

Obtain a Closed Loop Transfer Function

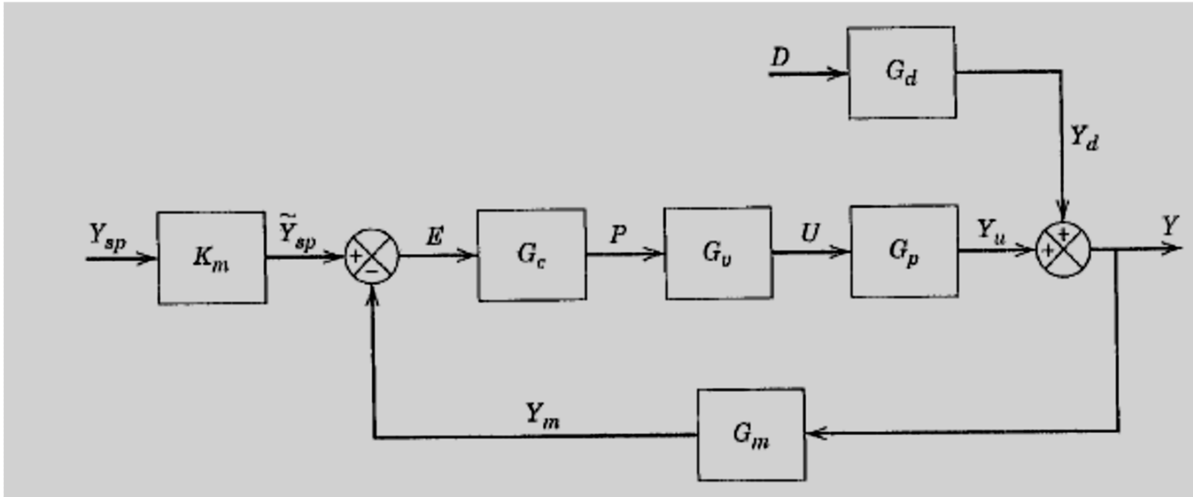


Figure 11.8 Standard block diagram of a feedback control system.

<u>Procedure</u>

1. Label All Signals
2. Write Equation for *Each* Signal
3. Substitute Equations to Solve for Y/Ysp and Y/D

Note: Shortcut (G_{cl} = Direct / (1+Loop)) method applicable to standard feedback system (Fig. 11.8)

Write Equation for Each Signal

$$\tilde{Y}_{sp} = K_m Y_{sp}$$

$$Y_m = G_m Y$$

$$E = \tilde{Y}_{sp} - Y_m$$

$$P = G_c E$$

$$U = G_v P$$

$$Y_u = G_p U$$

$$Y_d = G_d D$$

$$Y = Y_u + Y_d$$

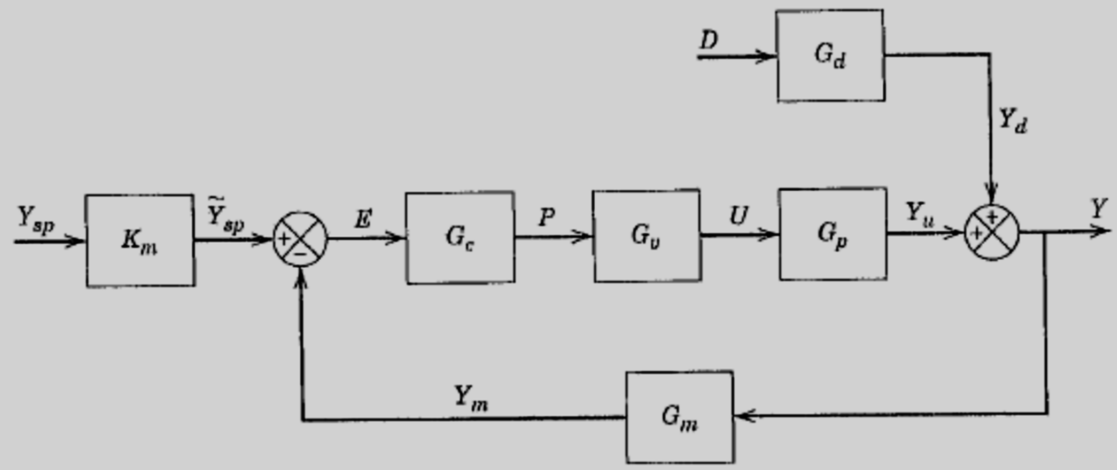


Figure 11.8 Standard block diagram of a feedback control system.

Substitute Equations

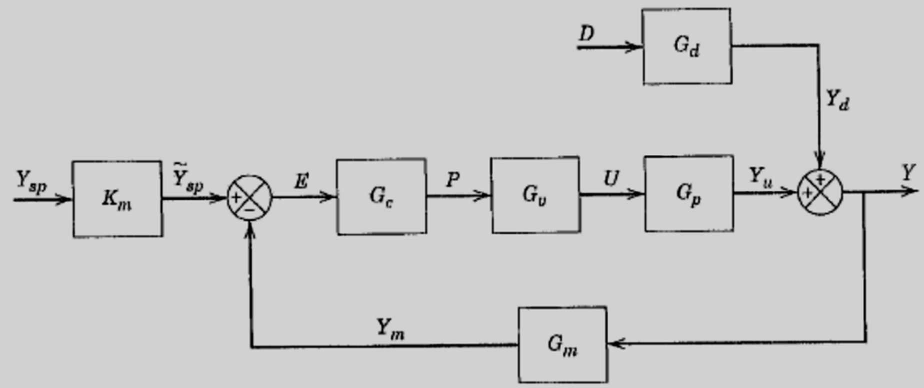


Figure 11.8 Standard block diagram of a feedback control system.

$$\tilde{Y}_{sp} = K_m Y_{sp}$$

$$Y_m = G_m Y$$

$$E = \tilde{Y}_{sp} - Y_m = K_m Y_{sp} - G_m Y$$

$$P = G_c E = G_c \left(K_m Y_{sp} - G_m Y \right)$$

$$U = G_v P = G_v G_c \left(K_m Y_{sp} - G_m Y \right)$$

$$Y_u = G_p U = G_p G_v G_c \left(K_m Y_{sp} - G_m Y \right)$$

$$Y_d = G_d D$$

$$Y = Y_u + Y_d = G_p G_v G_c \left(K_m Y_{sp} - G_m Y \right) + G_d D$$

Solve for Y/Ysp and Y/D

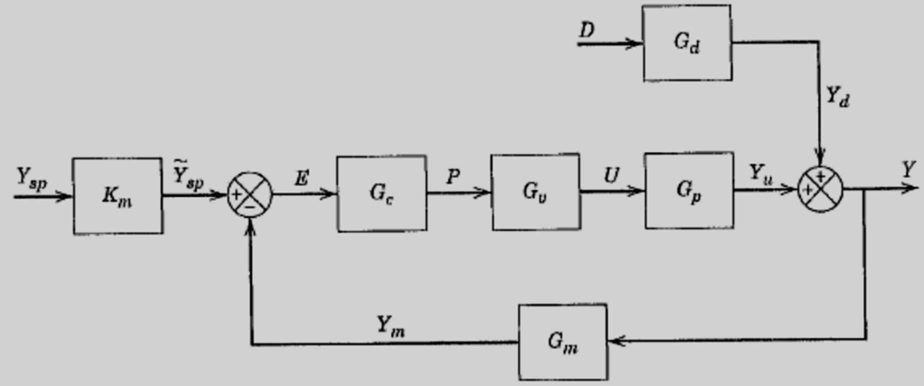


Figure 11.8 Standard block diagram of a feedback control system.

$$Y = G_p G_v G_c \left(K_m Y_{sp} - G_m Y\right) + G_d D$$

$$Y + G_p G_v G_c G_m Y = G_p G_v G_c K_m Y_{sp} + G_d D$$

$$Y(1 + G_p G_v G_c G_m) = G_p G_v G_c K_m Y_{sp} + G_d D$$

$$Y = \frac{G_p G_v G_c K_m}{1 + G_p G_v G_c G_m} Y_{sp} + \frac{G_d}{1 + G_p G_v G_c G_m} D$$

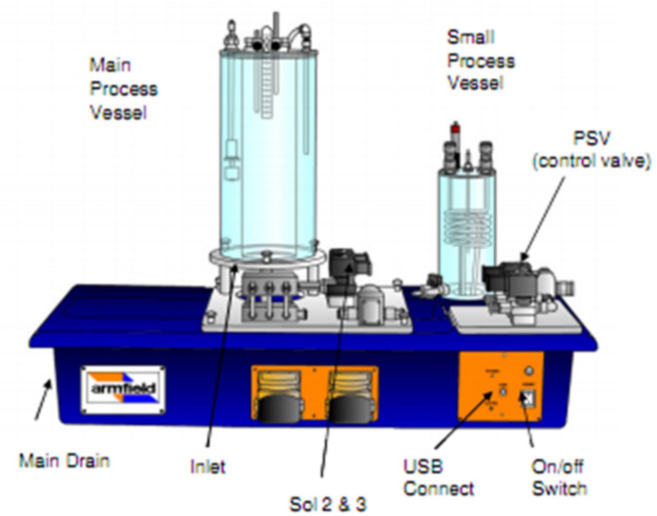
Control of a Gravity Drained Tank

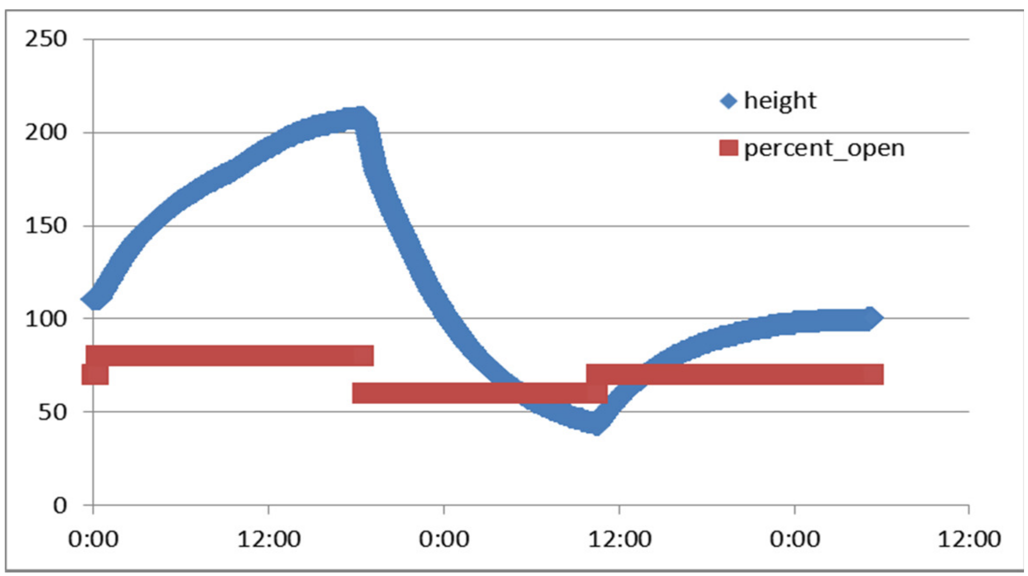
- Lab Experiment #1
- Material Balance

$$\frac{\partial m}{dt} = \dot{m}_{in} - \dot{m}_{out}$$

$$\rho A \frac{\partial h}{dt} = \rho\left(\dot{V}_{in} - \dot{V}_{out}\right)$$

$$A \frac{\partial h}{dt} = \dot{V}_{in} - \dot{V}_{out}$$





Source: Lee Jacobsen / James Memmott

Model of a Gravity Drained Tank

$$A\frac{\partial h}{dt} = \dot{V}_{in} - \dot{V}_{out}$$

$$A\frac{\partial h}{dt} = c_1(\%Open) - c_2\sqrt{h}$$

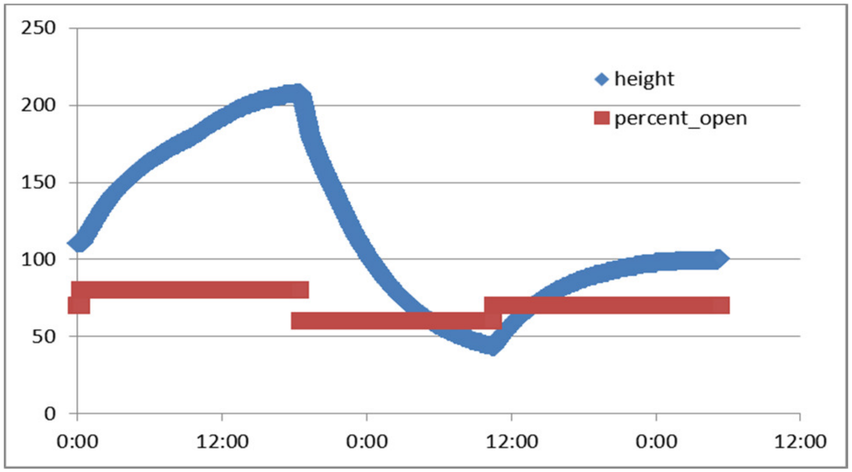
Linearize

$$A\frac{\partial h'}{dt} = c_1(\%Open') - c_2\left(\frac{1}{2}\bar{h}^{\left(-\frac{1}{2}\right)}\right)h'$$

$$A\frac{\partial h'}{dt} = c_1(\%Open') - c_3 h'$$

$$AsH(s) = c_1 O(s) - c_3 H(s)$$

$$\frac{H(s)}{O(s)} = \frac{c_1}{As + c_3} = \frac{c_1/c_3}{(A/c_3)s + 1} = \frac{K_p}{\tau_p s + 1}$$



Controller Block Diagram

$$K_m = 1$$

$$G_c = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

$$G_v = 1$$

$$G_p = \frac{K_p}{\tau_p s + 1}$$

$$G_m = 1$$

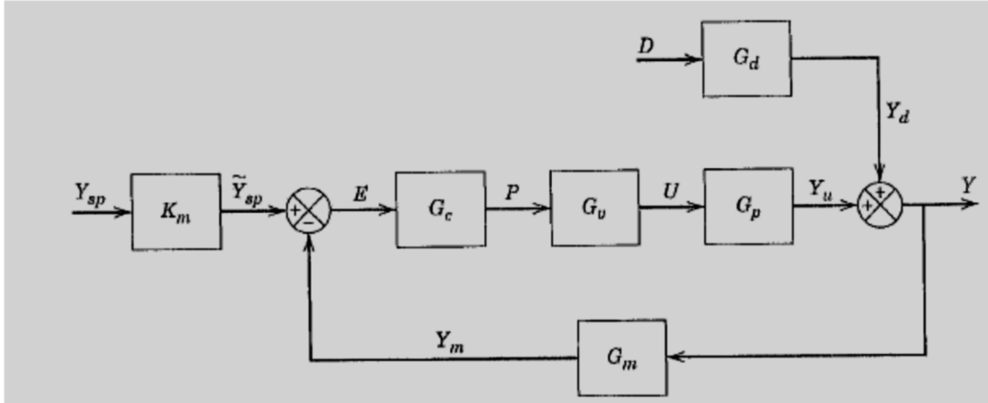
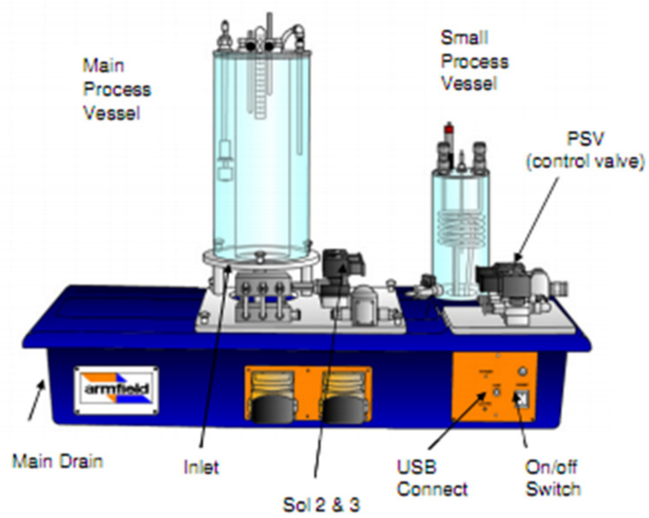


Figure 11.8 Standard block diagram of a feedback control system.



Closed Loop Transfer Function

$$K_m = 1$$

$$G_c = K_c\left(1 + \frac{1}{\tau_I s}\right)$$

$$G_v = 1$$

$$G_p = \frac{K_p}{\tau_p s + 1}$$

$$G_m = 1$$

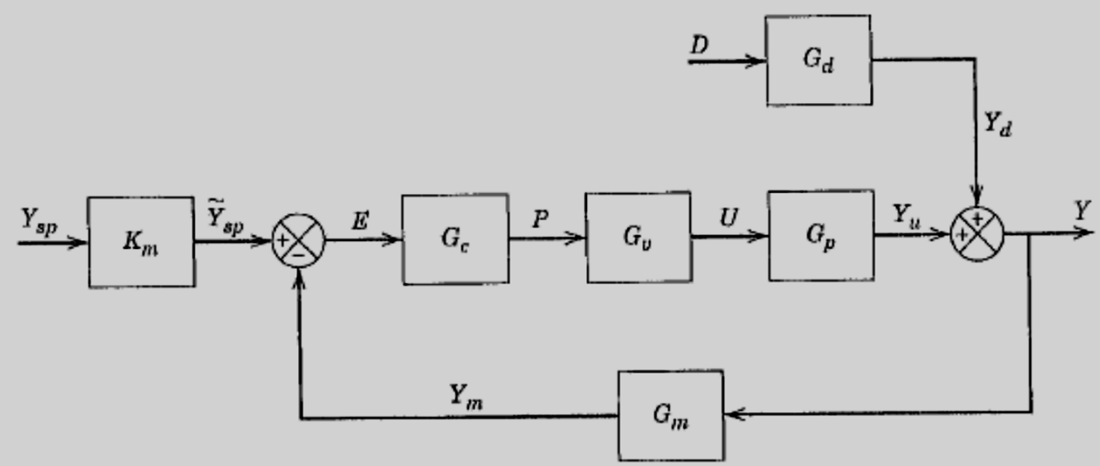


Figure 11.8 Standard block diagram of a feedback control system.

$$\frac{Y(s)}{Y_{sp}(s)} = G_{cl} = \frac{K_c\left(1 + \frac{1}{\tau_I s}\right)\frac{K_p}{\tau_p s + 1}}{\tau_I s\left(\tau_p s + 1\right) + K_c\left(1 + \frac{1}{\tau_I s}\right)\frac{K_p}{\tau_p s + 1}} = \frac{K_p K_c\left(\tau_I s + 1\right)}{\tau_I s\left(\tau_p s + 1\right) + K_p K_c\left(\tau_I s + 1\right)}$$

Closed Loop Transfer Function

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{K_p K_c (\tau_I s + 1)}{\tau_I s(\tau_p s + 1) + K_p K_c (\tau_I s + 1)}$$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{K_p K_c (\tau_I s + 1)}{\tau_I \tau_p s^2 + (1 + K_p K_c)\tau_I s + K_p K_c}$$

Standard Form

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{\tau_I s + 1}{\frac{\tau_p \tau_I}{K_p K_c} s^2 + \left(\frac{1}{K_p K_c} + 1\right)\tau_I s + 1}$$

$$\tau_{cl} = \sqrt{\frac{\tau_p \tau_I}{K_p K_c}}, \quad 2\zeta_{cl}\tau_{cl} = \left(\frac{1}{K_p K_c} + 1\right)\tau_I, \quad \zeta_{cl} = \frac{\tau_I (K_p K_c + 1)}{2 K_p K_c \sqrt{\frac{\tau_p \tau_I}{K_p K_c}}}$$