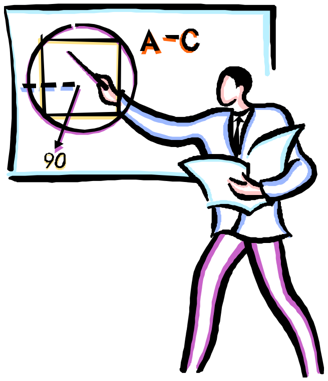
Class 31

Stability Analysis



Stability Analysis

- Stability with changing PID tuning
  - What is the range of K_c that gives a stable system?

<u>Techniques for Stability Analysis</u>

- Routh Array
- Root Locus Plot
- Bode Diagram
- Not covered in this lecture
  - Direct substitution
  - Nyquist diagram

Example 11.13 (modified)

Consider a feedback control system that has the open-loop transfer function,

$$G_{OL}(s) = \frac{4K_c}{(s+1)(s+2)(s+3)} \qquad (11\text{-}108)$$

Determine the values of K_c that keep the closed loop system response stable.

Part A) What is the denominator of closed loop response (Characteristic Equation)?

Characteristic Equation

The characteristic equation is $1 + G_{OL} = 0$ or

$$(s+1)(s+2)(s+3)+4K_c=0 \tag{11-109}$$

Routh Array

$$a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0 \qquad (11\text{-}93)$$

$a_n > 0$ Multiply polynomial by -1 if $a_n<0$

Row				
1	a_n	a_{n-2}	a_{n-4}	$\cdots$
2	a_{n-1}	a_{n-3}	a_{n-5}	$\cdots$
3	b_1	b_2	b_3	$\cdots$
4	c_1	c_2	$\cdots$	
$\vdots$	$\vdots$			
$n+1$	z_1			

Stable if leading edge is positive

$$b_1 = \frac{a_{n-1}a_{n-2} - a_n a_{n-3}}{a_{n-1}} \qquad (11\text{-}94)$$

$$b_2 = \frac{a_{n-1}a_{n-4} - a_n a_{n-5}}{a_{n-1}} \qquad (11\text{-}95)$$

$$\vdots$$

$$c_1 = \frac{b_1 a_{n-3} - a_{n-1} b_2}{b_1} \qquad (11\text{-}96)$$

$$c_2 = \frac{b_1 a_{n-5} - a_{n-1} b_3}{b_1} \qquad (11\text{-}97)$$

<u>Advantages</u>

- Quick Stable vs. Unstable
- No calculator / computer required

<u>Disadvantages</u>

- No indication of stability margin
- Polynomial Characteric Equations only, No time-delays

Part B) For example problem, calculate the critical gain, K_{cu}

Routh Array Example

$$(s+1)(s+2)(s+3)+4K_c=0 \qquad (11\text{-}109)$$

$$s^3+6s^2+11s+6+4K_c=0$$

Routh Array:

$$1 \quad 11$$

$$6 \quad (6+4K_c)$$

$$b_1=\frac{a_{n-1}a_{n-2}-a_n a_{n-3}}{a_{n-1}}=\frac{6\cdot 11-1(6+4K_c)}{6}$$

Leading Edge > 0:

$$a_n>0$$

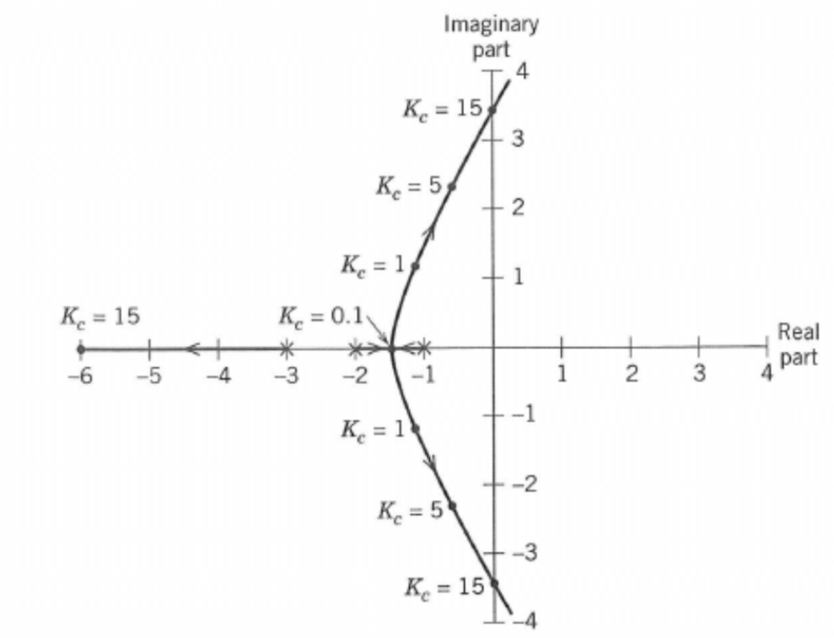
$$a_{n-1}>0$$

$$b_1>0, \quad \frac{6\cdot 11-1(6+4K_c)}{6}>0, \quad K_c<15$$

Root Locus Diagrams

- Find characteristic equation
  - Equation is a function of K_c
- Pick various values of K_c
- Find roots of characteristic equation
- Plot on real vs. imaginary coordinate system

Root Locus Diagrams



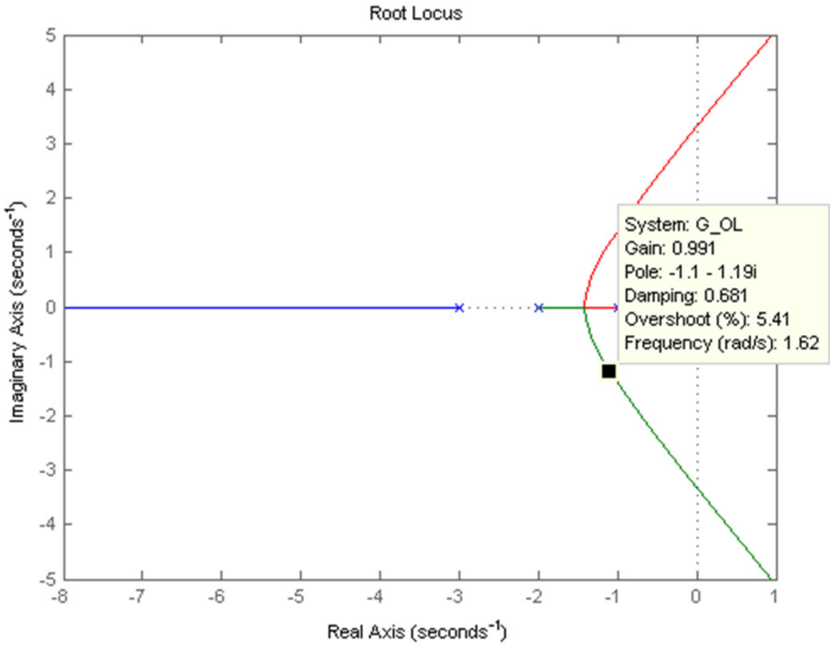
- Part c) For example problem, calculate the critical gain, K_{cu}
- The root locus diagram in Fig. 11.27 shows how the three roots of this characteristic equation vary with K_c.
- When $K_c = 0$, the roots are merely the poles of the open-loop transfer function, -1, -2, and -3.

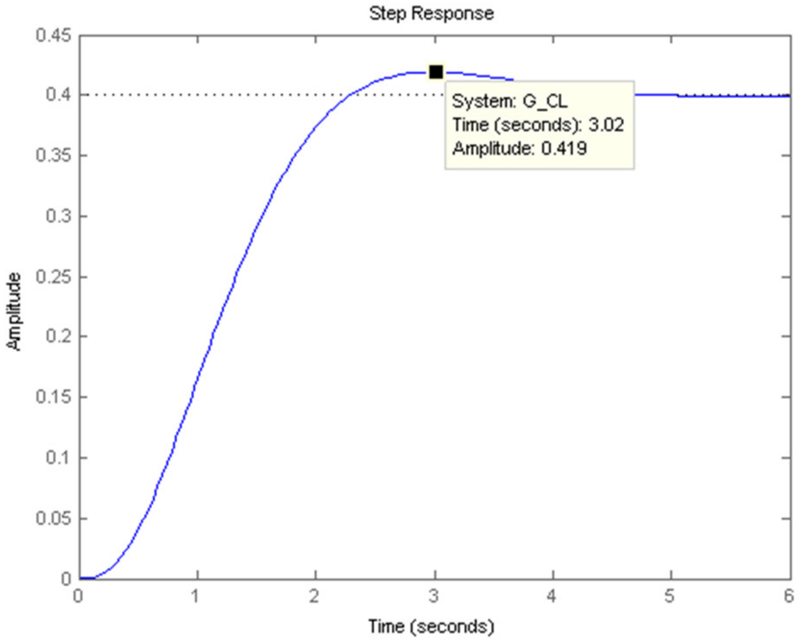
Figure 11.27 Root locus diagram for third-order system. X denotes an open-loop pole. Dots denote locations of the closed-loop poles for different values of K_c. Arrows indicate change of pole locations as K_c increases.

See MATLAB Commands Sheet

Root Locus Diagram - Overshoot

- With K_c=1, Overshoot = 5%





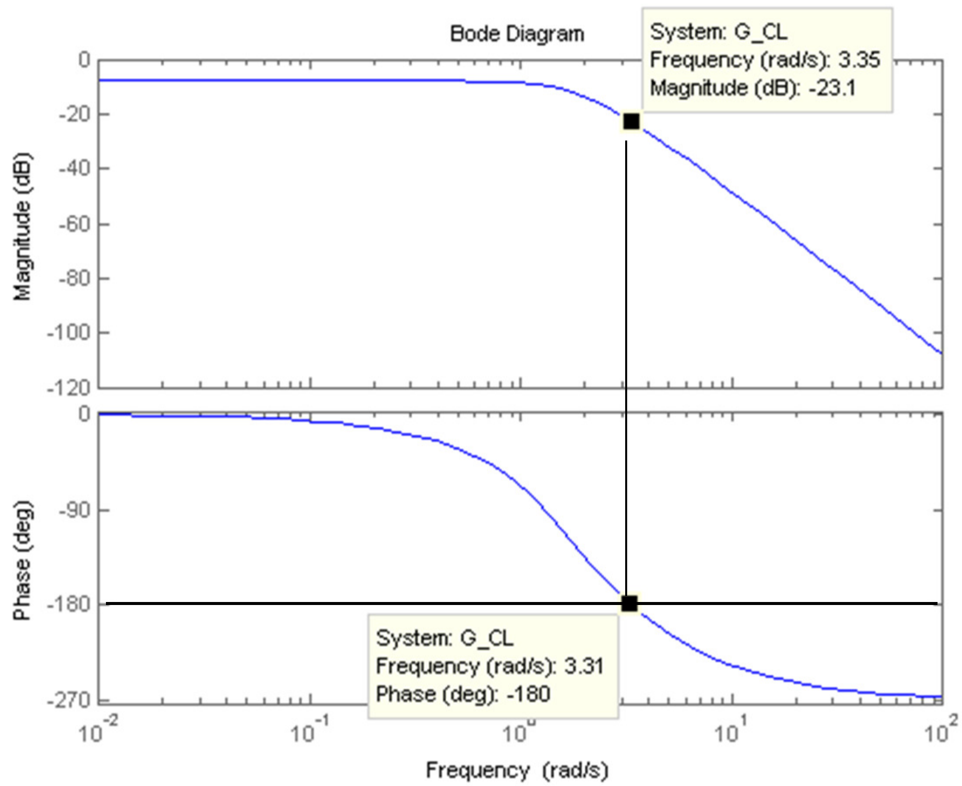
Bode Plot Stability Analysis

- Critical frequency ω_c is ω for which $\phi_{OL}(\omega)$=-180°
- Amplitude Ratio at Critical Frequency
  - Stable when $AR_{OL}(\omega_c) < 1$
- Decibels to Amplitude Ratio
  - Decibels = G_{dB} = 20 $\log_{10}(AR)$
  - Stability: $AR < 1$ or $G_{dB} < 0$
- Gain Margin

$$K_{cu} = \frac{1}{AR_G(\omega_c)} = \frac{1}{10^{\frac{G_{dB}}{20}}}$$

- Part d) For example problem calculate K_{cu} using the Bode diagram.

Bode Plot Stability Analysis



$$K_{cu} = \frac{1}{AR_G(\omega_c)} = \frac{1}{10^{\frac{G_{dB}}{20}}} = \frac{1}{10^{-\frac{23.1}{20}}} = 14.3 \approx 15$$