

**Table 3.1** Laplace Transforms for Various Time-Domain Functions<sup>a</sup>

$f(t)$	$F(s)$
1. $\delta(t)$ (unit impulse)	1
2. $S(t)$ (unit step)	$\frac{1}{s}$
3. $t$ (ramp)	$\frac{1}{s^2}$
4. $t^{n-1}$	$\frac{(n-1)!}{s^n}$
5. $e^{-bt}$	$\frac{1}{s+b}$
6. $\frac{1}{\tau}e^{-t/\tau}$	$\frac{1}{\tau s+1}$
7. $\frac{t^{n-1}e^{-bt}}{(n-1)!}$ ( $n > 0$ )	$\frac{1}{(s+b)^n}$
8. $\frac{1}{\tau^n(n-1)!}t^{n-1}e^{-t/\tau}$	$\frac{1}{(\tau s+1)^n}$
9. $\frac{1}{b_1 - b_2}(e^{-b_2 t} - e^{-b_1 t})$	$\frac{1}{(s+b_1)(s+b_2)}$
10. $\frac{1}{\tau_1 - \tau_2}(e^{-t/\tau_1} - e^{-t/\tau_2})$	$\frac{1}{(\tau_1 s+1)(\tau_2 s+1)}$
11. $\frac{b_3 - b_1}{b_2 - b_1}e^{-b_1 t} + \frac{b_3 - b_2}{b_1 - b_2}e^{-b_2 t}$	$\frac{s+b_3}{(s+b_1)(s+b_2)}$
12. $\frac{1}{\tau_1}\frac{\tau_1 - \tau_3}{\tau_1 - \tau_2}e^{-t/\tau_1} + \frac{1}{\tau_2}\frac{\tau_2 - \tau_3}{\tau_2 - \tau_1}e^{-t/\tau_2}$	$\frac{\tau_3 s+1}{(\tau_1 s+1)(\tau_2 s+1)}$
13. $1 - e^{-t/\tau}$	$\frac{1}{s(\tau s+1)}$
14. $\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
15. $\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
16. $\sin(\omega t + \phi)$	$\frac{\omega \cos \phi + s \sin \phi}{s^2 + \omega^2}$
17. $e^{-bt} \sin \omega t$	$\left. \frac{\omega}{(s+b)^2 + \omega^2} \right\}$
18. $e^{-bt} \cos \omega t$	$\left. \frac{s+b}{(s+b)^2 + \omega^2} \right\}$
19. $\frac{1}{\tau \sqrt{1-\zeta^2}} e^{-\zeta t/\tau} \sin(\sqrt{1-\zeta^2} t/\tau)$ $(0 \leq  \zeta  < 1)$	$\frac{1}{\tau^2 s^2 + 2\zeta \tau s + 1}$
20. $1 + \frac{1}{\tau_2 - \tau_1}(\tau_1 e^{-t/\tau_1} - \tau_2 e^{-t/\tau_2})$ $(\tau_1 \neq \tau_2)$	$\frac{1}{s(\tau_1 s+1)(\tau_2 s+1)}$
21. $1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta t/\tau} \sin[\sqrt{1-\zeta^2} t/\tau + \psi]$ $\psi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}, (0 \leq  \zeta  < 1)$	$\frac{1}{s(\tau^2 s^2 + 2\zeta \tau s + 1)}$
22. $1 - e^{-\zeta t/\tau} [\cos(\sqrt{1-\zeta^2} t/\tau) + \frac{\zeta}{\sqrt{1-\zeta^2}} \sin(\sqrt{1-\zeta^2} t/\tau)]$ $(0 \leq  \zeta  < 1)$	$\frac{1}{s(\tau^2 s^2 + 2\zeta \tau s + 1)}$

**Table 3.1 (Continued)**

$f(t)$	$F(s)$
23. $1 + \frac{\tau_3 - \tau_1}{\tau_1 - \tau_2} e^{-t/\tau_1} + \frac{\tau_3 - \tau_2}{\tau_2 - \tau_1} e^{-t/\tau_2}$ $(\tau_1 \neq \tau_2)$	$\frac{\tau_3 s + 1}{s(\tau_1 s + 1)(\tau_2 s + 1)}$
24. $\frac{df}{dt}$	$sF(s) - f(0)$
25. $\frac{d^n f}{dt^n}$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f^{(1)}(0) - \dots - s f^{(n-2)}(0) - f^{(n-1)}(0)$
26. $f(t - t_0)S(t - t_0)$	$e^{-t_0 s} F(s)$

<sup>a</sup>Note that  $f(t)$  and  $F(s)$  are defined for  $t \geq 0$  only.