Dynamic Process Modeling

Understanding Dynamic Process Behavior

- To learn about the dynamic behavior of a process, analyze measured process variable test data
- Process variable test data can be generated by suddenly changing the controller output signal
- Be sure to move the controller output far enough and fast enough so that the dynamic behavior of the process is clearly revealed as the process responds
- The dynamic behavior of a process is different as operating level changes (nonlinear behavior) so collect process data at normal operating levels (design level of operation)

Modeling Dynamic Process Behavior

- The best way to understand process data is through modeling
- Modeling means fitting a first order plus dead time (FOPDT) dynamic process model to the data set:

$$\tau_P \frac{dy(t)}{dt} + y(t) = K_P\, u(t - \theta_P)$$

where:

$y(t)$ is the measured process variable

$u(t)$ is the controller output signal

- The FOPDT model is low order and linear so it can only approximate the behavior of real processes

Modeling Dynamic Process Behavior

- When a first order plus dead time (FOPDT) model is fit to dynamic process data

$$\tau_P \frac{dy(t)}{dt} + y(t) = K_P u(t - \theta_P)$$

- The important parameters that result are:
    - Steady State Process Gain, K_P
    - Overall Process Time Constant, τ_P
    - Apparent Dead Time, θ_P

PID Tuning Guide

Begin by fitting a first order plus dead time (FOPDT) dynamic model to process data. "Process" is defined to include all dynamic information from the output signal of the controller through the measured response signal of the process variable.

Generate process data by forcing the measured process variable with a change in the controller output signal. For accurate results:

- the process must begin at steady state; the first data point recorded to file must equal that steady state value
- the data collection sample rate should be ten times per time constant or faster ($T \le 0.1\,\tau_P$)
- the controller output should force the measured process variable to move at least ten times the noise band

Use *Design Tools* to fit a FOPDT dynamic model to the process data set. A FOPDT model has the form:

Time Domain: $\tau_P \frac{dy(t)}{dt} + y(t) = K_P\, u(t - \theta_P)$ **Laplace Domain**: $\frac{Y(s)}{U(s)} = \frac{K_P e^{-\theta_P s}}{\tau_P s + 1}$

where:
- $y(t)$ = measured process variable signal
- $u(t)$ = controller output signal
- K_P = process gain; units of $y(t)/u(t)$
- τ_P = process time constant; units of time
- θ_P = process dead time; units of time

also:
- K_C = controller gain; units of $u(t)/y(t)$
- τ_I = controller reset time; units of time
- τ_D = controller derivative time; units of time
- α = derivative filter constant; unitless

Values of K_P, τ_P and θ_P that describe the dynamic behavior of your process are important because:

- they are used in correlations (listed below) to compute initial PID controller tuning values K_C, τ_I, τ_D and α
- the sign of K_P indicates the action of the controller ($+K_P \rightarrow$ reverse acting; $-K_P \rightarrow$ direct acting)
- the size of τ_P indicates the maximum desirable loop sample time (be sure sample time $T \le 0.1\,\tau_P$)
- the ratio θ_P/τ_P indicates whether a Smith predictor would show benefit (useful when $\theta_P/\tau_P > 0.7$)
- the model itself is used in feed forward, Smith predictor, decoupling and other model-based controllers

These correlations provide a starting point for tuning. Final tuning requires online trial and error. "Best" tuning is defined by you and your knowledge of the capabilities of the process, desires of management, goals of production, and impact on other processes.

IMC (lambda) Tuning

Standard Tuning: τ_C is the larger of $0.1\tau_P$ or $0.8\theta_P$

Conservative Tuning: τ_C is the larger of $0.5\tau_P$ or $4.0\theta_P$

* This is an ITAE correlation as no P-Only IMC exists

	K_C	τ_I	τ_D	α
P-Only*	$\frac{0.202}{K_P}(\theta_P/\tau_P)^{-1.219}$			
PI	$\frac{1}{K_P}\frac{\tau_P}{(\theta_P+\tau_C)}$	τ_P		
PID Ideal	$\frac{1}{K_P}\left(\frac{\tau_P+0.5\theta_P}{\tau_C+0.5\theta_P}\right)$	$\tau_P+0.5\theta_P$	$\frac{\tau_P\theta_P}{2\tau_P+\theta_P}$	
PID Interacting	$\frac{1}{K_P}\left(\frac{\tau_P}{\tau_C+0.5\theta_P}\right)$	τ_P	$0.5\theta_P$	
PID Ideal w/filter	$\frac{1}{K_P}\left(\frac{\tau_P+0.5\theta_P}{\tau_C+\theta_P}\right)$	$\tau_P+0.5\theta_P$	$\frac{\tau_P\theta_P}{2\tau_P+\theta_P}$	$\frac{\tau_C(\tau_P+0.5\theta_P)}{\tau_P(\tau_C+\theta_P)}$
PID Interacting w/filter	$\frac{1}{K_P}\left(\frac{\tau_P}{\tau_C+\theta_P}\right)$	τ_P	$0.5\theta_P$	$\frac{\tau_C}{\tau_C+\theta_P}$

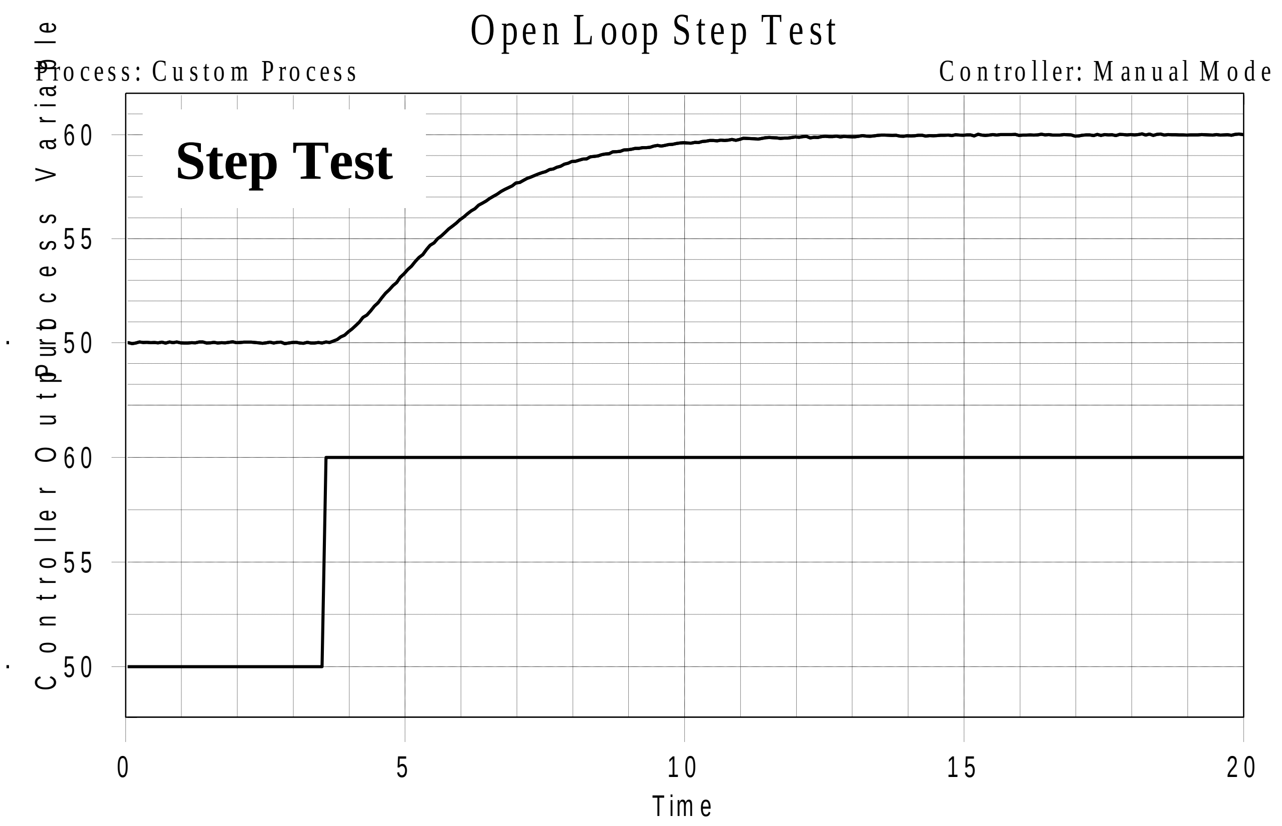
- Tuning of controller means selecting the parameters for "best" operation
- FOPDT model of dynamics of the process are used
  - K_p
  - τ_p
  - θ_p
- See correlations
- This "Tuning Guide" is located near the end of the pdf file for the Practical Process Control book
- This is a place to start (close but not best)

The FOPDT Model

- model parameters (K_P, τ_P and θ_P) are used in correlations to compute initial controller tuning values
- sign of K_P indicates the action of the controller

  ($+K_P \rightarrow$ reverse acting; $-K_P \rightarrow$ direct acting)
- size of τ_P indicates the maximum desirable loop sample time (be sure sample time $T \leq 0.1\,\tau_P$)

Step Test Data and Dynamic Process Modeling



- Process starts at steady state
- Controller output signal is stepped to new value
- Measured process variable allowed to complete response

Process Gain From Step Test Data

- K_P describes how much the measured process variable, $y(t)$, changes in response to changes in the controller output, $u(t)$

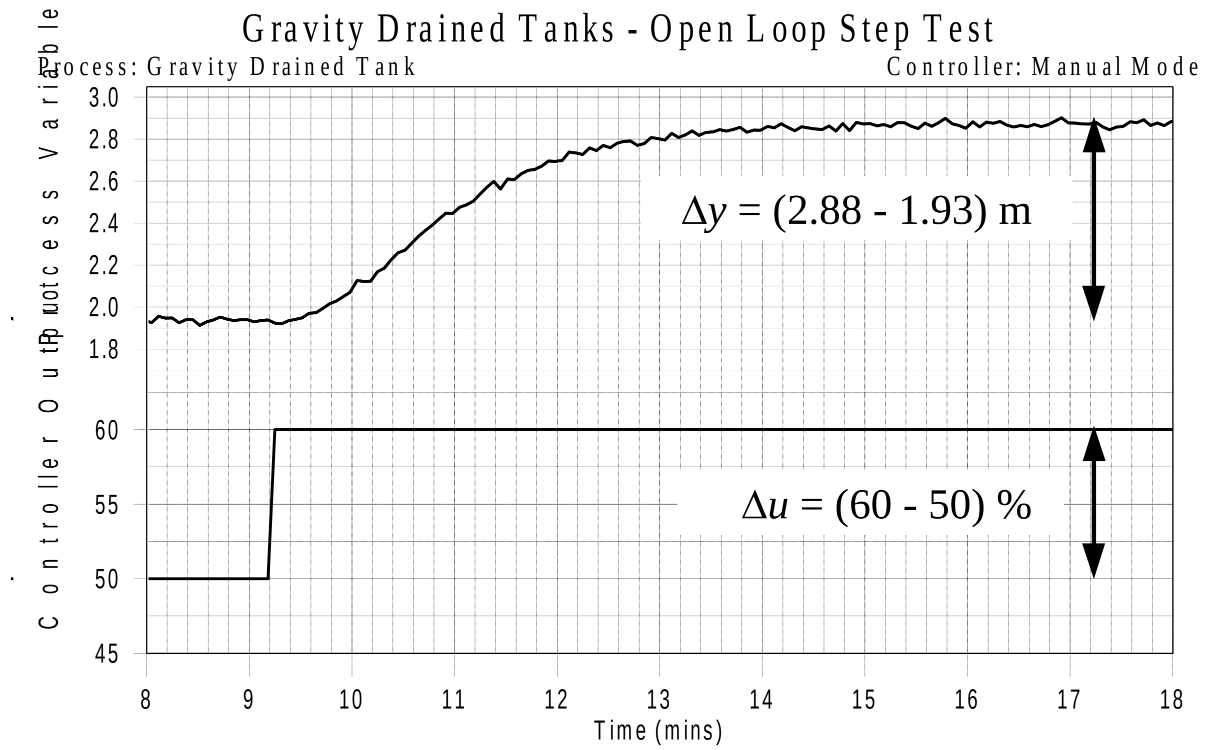
- A step test starts and ends at steady state, so K_P can be computed from plot axes

$$K_P = \frac{\text{Steady State Change in the Measured Process Variable, } \Delta y(t)}{\text{Steady State Change in the Controller Output, } \Delta u(t)}$$

where $\Delta u(t)$ and $\Delta y(t)$ represent the total change from initial to final steady state

- A large process gain means the process will show a big response to each control action

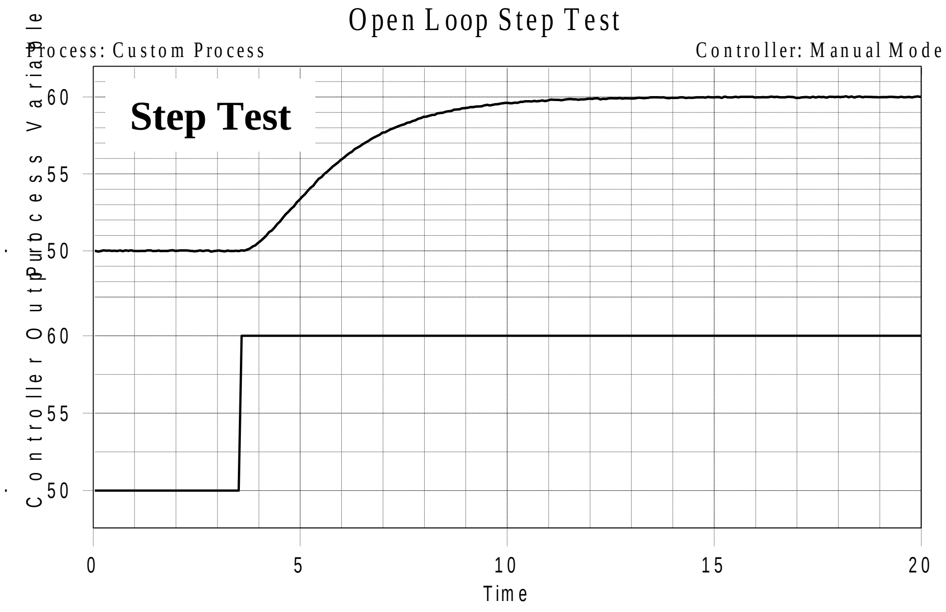
K_P for Gravity Drained Tanks



$$K_P = \frac{\Delta y}{\Delta u} = \frac{2.88 - 1.93\ \text{m}}{60 - 50\%} = 0.095\ \frac{\text{m}}{\%}$$

Steady state process gain has a:
size (0.095), sign (+0.095), and units (m/%)

Overall Time Constant From Step Test Data



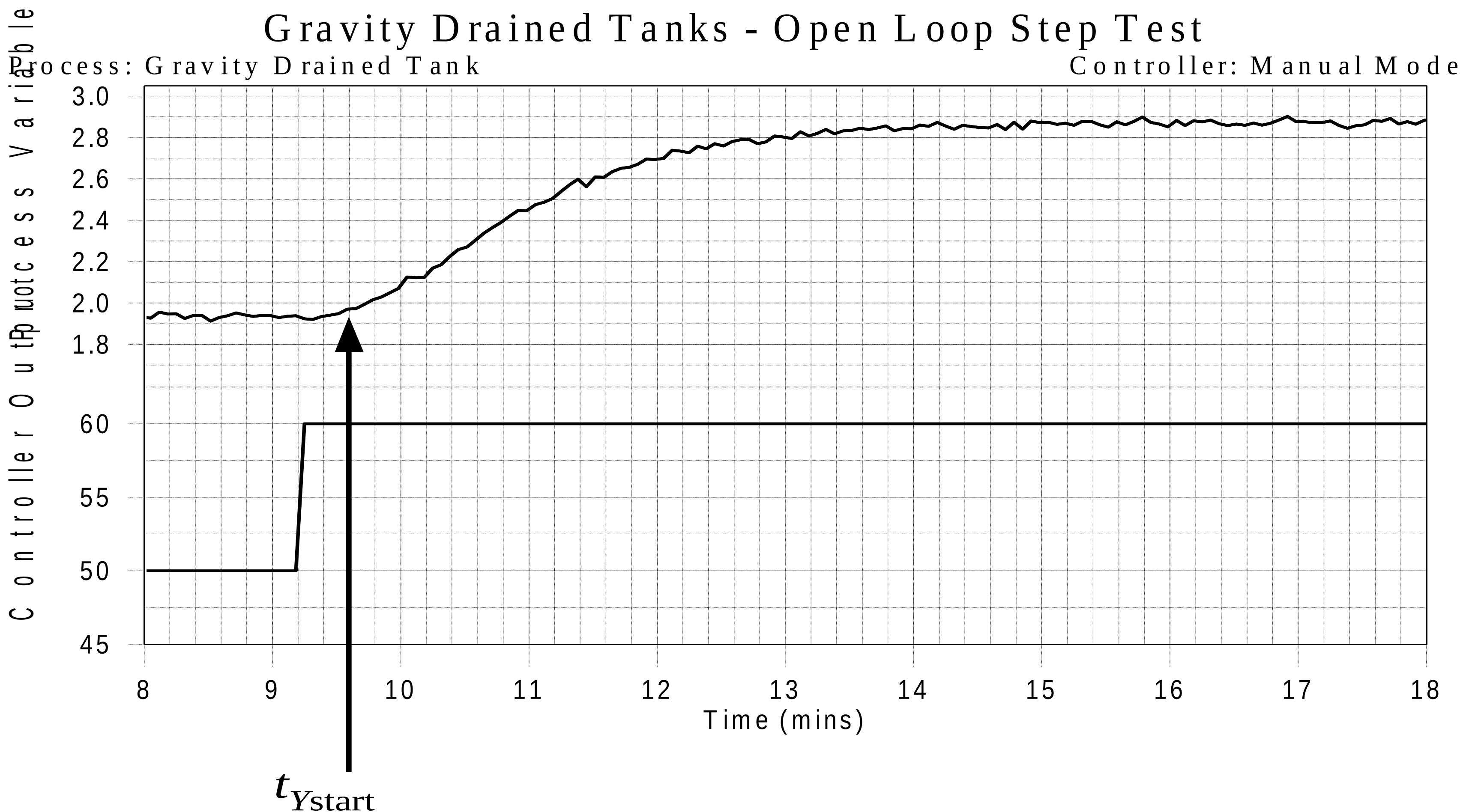
Time Constant τ_P describes how fast the measured process variable, $y(t)$, responds to changes in the controller output, $u(t)$

τ_P is how long it takes for the process variable to reach 63.2% of its total change, starting from when the response first begins

τ_P for Gravity Drained Tanks

1) Locate where the measured process variable first shows a clear initial response to the step change – call this time t_{Ystart}

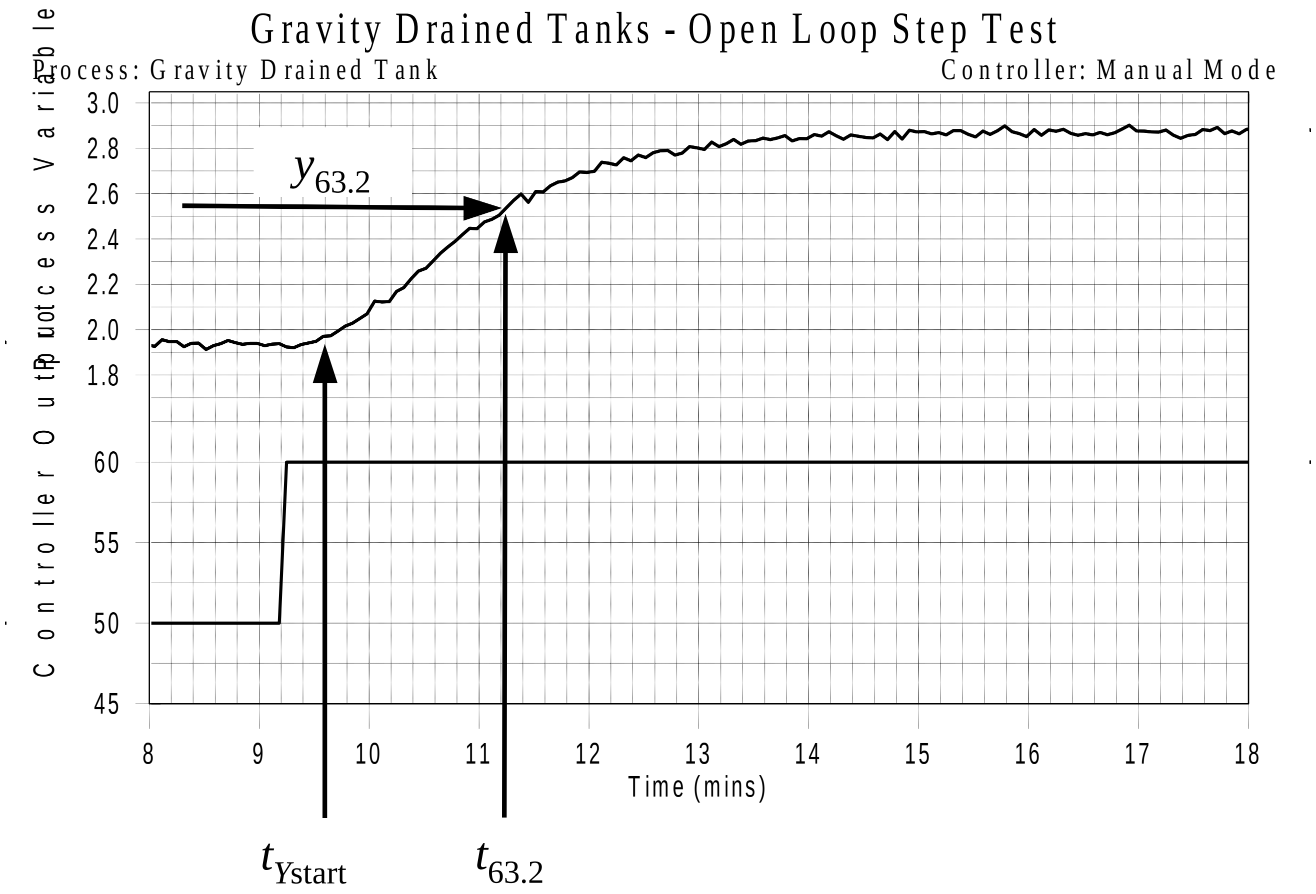
From plot, t_{Ystart} = 9.6 min



τ_P for Gravity Drained Tanks

2) Locate where the measured process variable reaches $y_{63.2}$, or where $y(t)$ reaches 63.2% of its total final change

Label time $t_{63.2}$ as the point in time where $y_{63.2}$ occurs



Why is tau = 63.2% to steady-state?

$$\tau \frac{\partial y}{\partial t} = -y + Ku \quad \text{drop the time - delay from FOPDT equation}$$

$$\tau sY(s) - y(0) = -Y(s) + KU(s) \quad \text{LaPlace Transform (pg. 42 of PDC Book)}$$

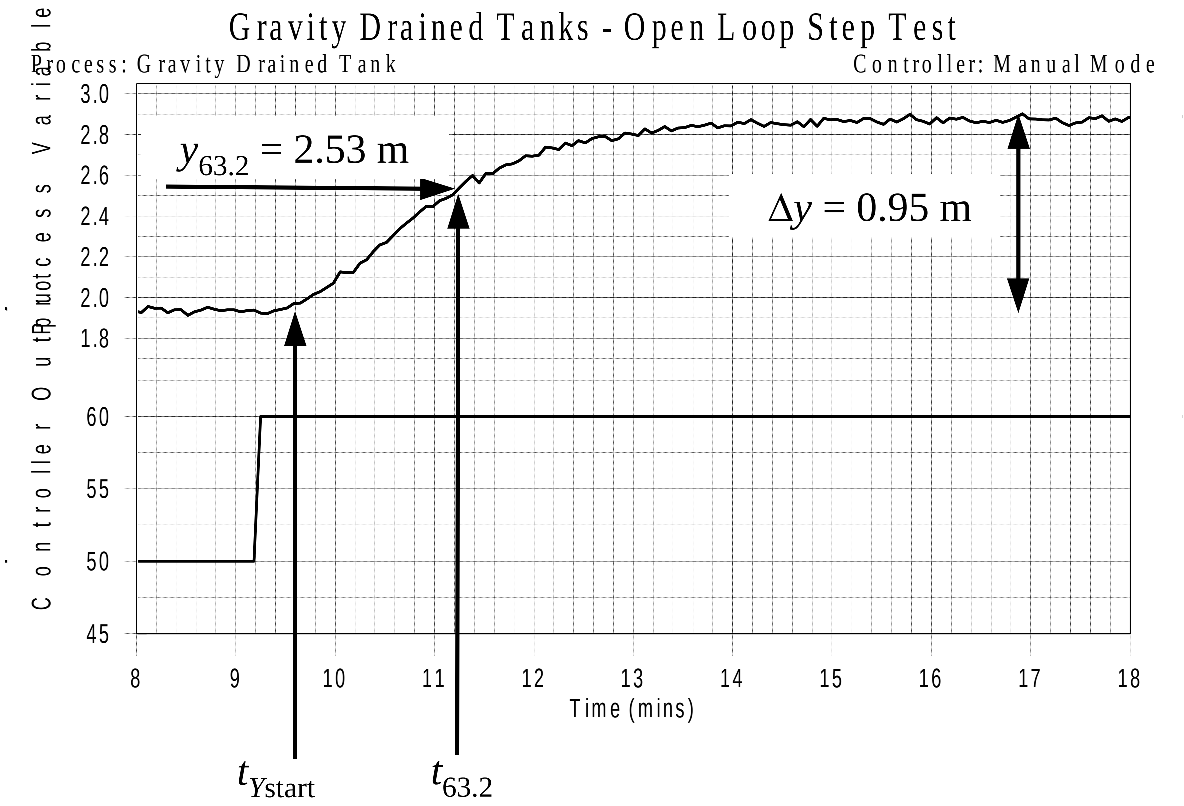
$$\frac{Y(s)}{U(s)} = \frac{K}{\tau s + 1} \quad \text{Rearrange with y(0) = 0 to obtain Transfer Function}$$

$$Y(s) = \frac{1}{s}\left(\frac{K}{\tau s + 1}\right) \quad \text{U(s) = 1/s (step function)}$$

$$y(t) = K\left(1 - e^{-\frac{t}{\tau}}\right) \quad \text{Inverse LaPlace Transform (pg.42 of PDC Book)}$$

$$y(t) = K\left(1 - e^{-1}\right) = K(0.632) \quad \text{At } t = \tau$$

τ_P for Gravity Drained Tanks



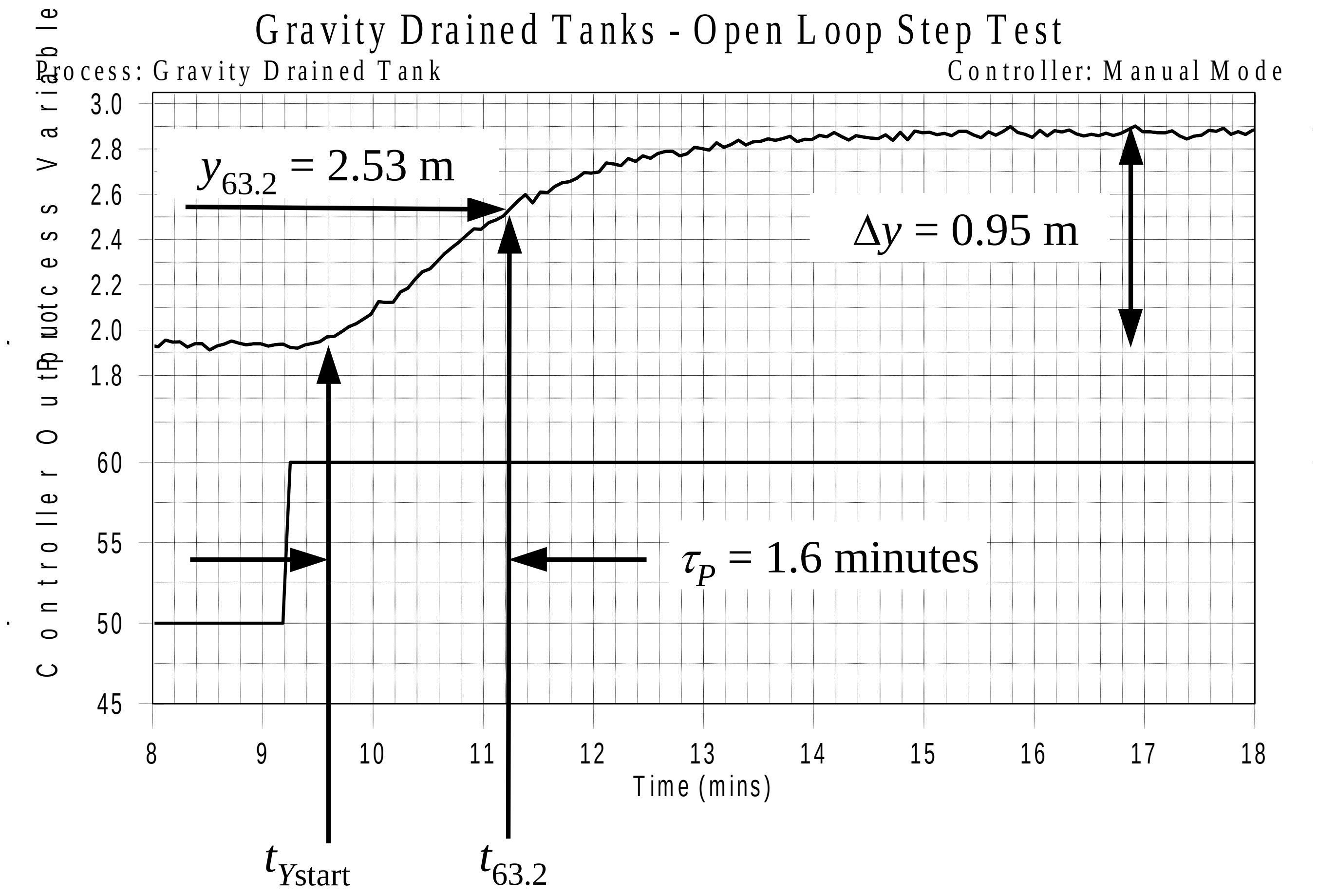
- $y(t)$ starts at 1.93 m and shows a total change Δy = 0.95 m
- $y_{63.2}$ = 1.93 m + 0.632(Δy)

  = 1.93 m + 0.632(0.95 m) = 2.53 m
- $y(t)$ passes through 2.53 m at $t_{63.2}$ = 11.2 min

τ_P for Gravity Drained Tanks

- The time constant is the time difference between t_{Ystart} and $t_{63.2}$
- Time constant must be positive and have units of time

From the plot: $\tau_P = t_{63.2} - t_{Ystart} = 11.2 \text{ min} - 9.6 \text{ min} = 1.6 \text{ min}$

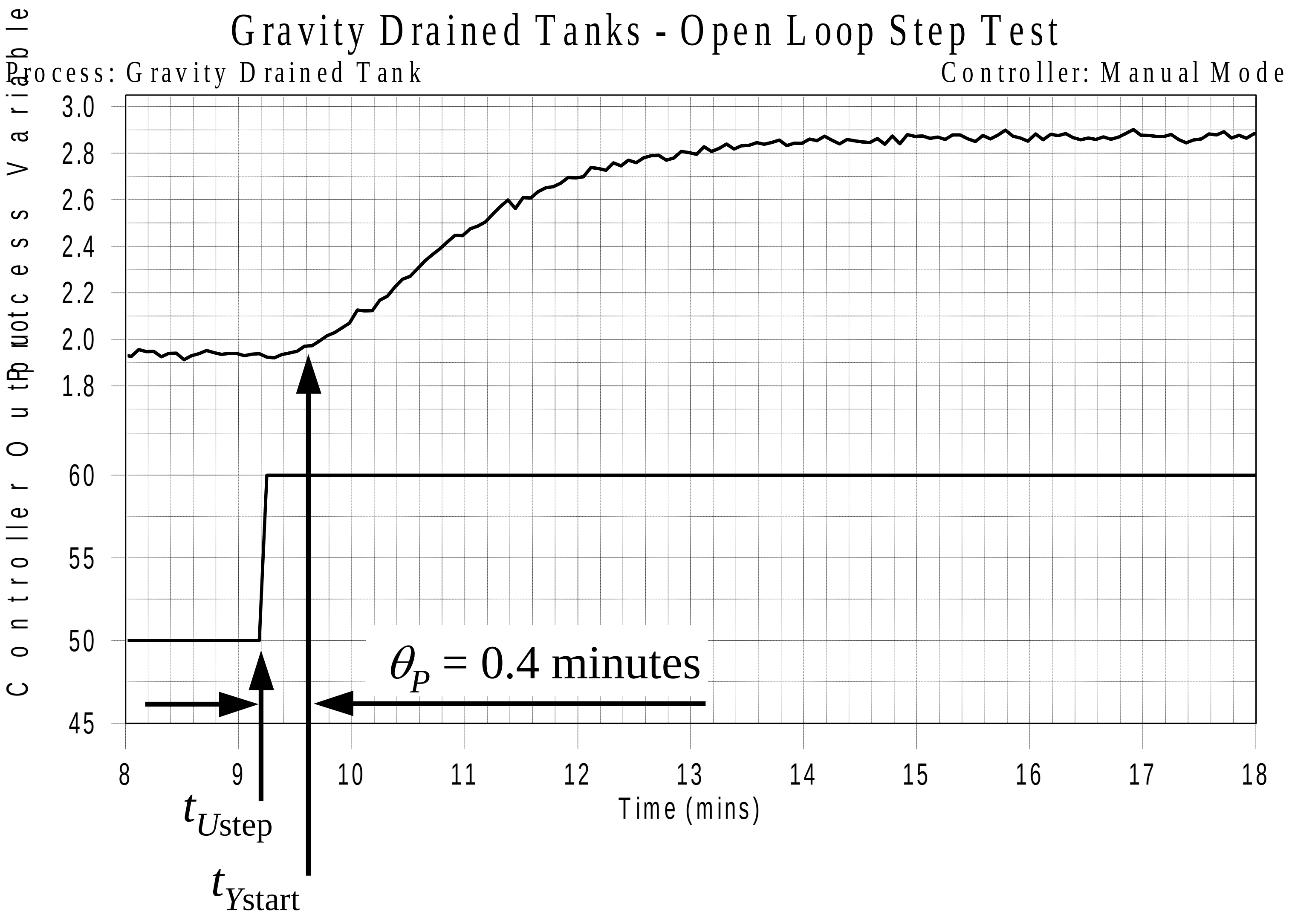


Apparent Dead Time From Step Test Data

- θ_P is the time from when the controller output step is made until when the measured process variable first responds
- Apparent dead time, θ_P, is the sum of these effects:
  - transportation lag, or the time it takes for material to travel from one point to another
  - sample or instrument lag, or the time it takes to collect analyze or process a measured variable sample
  - higher order processes naturally appear slow to respond

- Notes:
  - Dead time must be positive and have units of time
  - Tight control in increasingly difficult as $\theta_P > 0.7\,\tau_P$
  - For important loops, work to avoid unnecessary dead time

θ_P for Gravity Drained Tanks



$$\theta_P = t_{Ystart} - t_{Ustep}$$
$$= 9.6 \text{ min} - 9.2 \text{ min}$$
$$= 0.4 \text{ min}$$

Processes Have Time-Varying Behaviors

- The predictions of a FOPDT model are constant over time

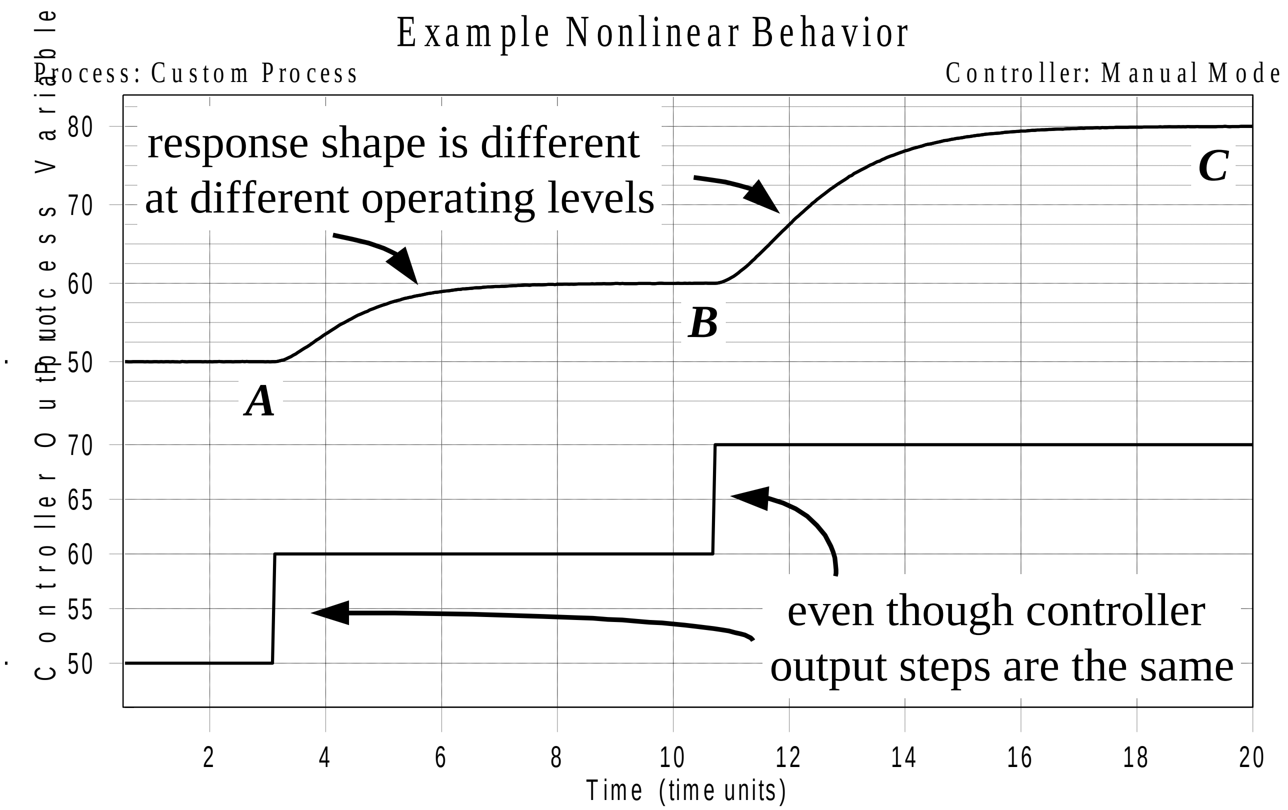
- But real processes change every day because
  - surfaces foul or corrode
  - mechanical elements like seals or bearings wear
  - feedstock quality varies and catalyst activity drifts
  - environmental conditions like heat and humidity change

- So the values of K_P, τ_P, θ_P that best describe the dynamic behavior of a process today may not be best tomorrow

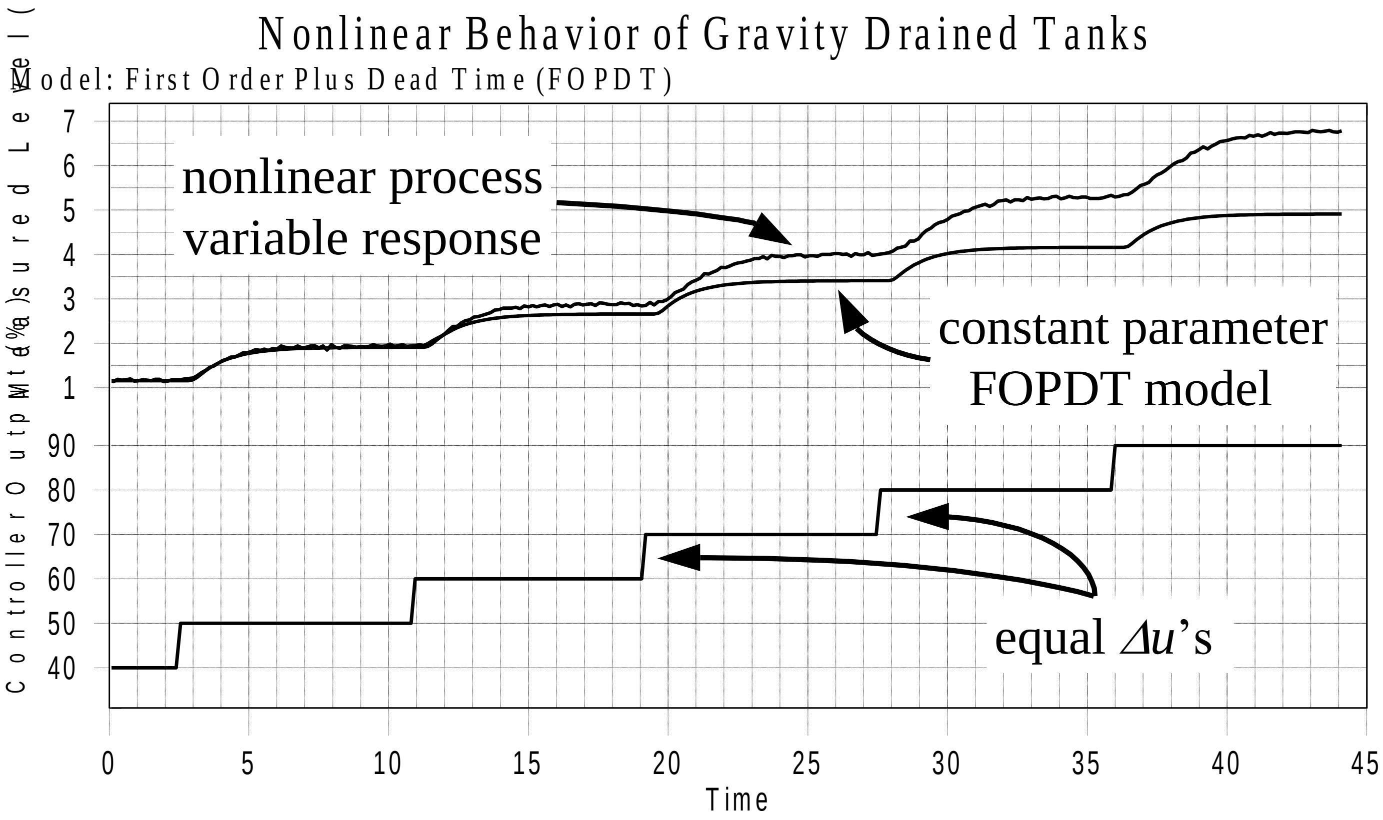
- As a result, controller performance will degrade with time and periodic retuning may be required

Processes Have Nonlinear Behaviors

- The predictions of a FOPDT model are constant as operating level changes
- The response of a real process varies with operating level



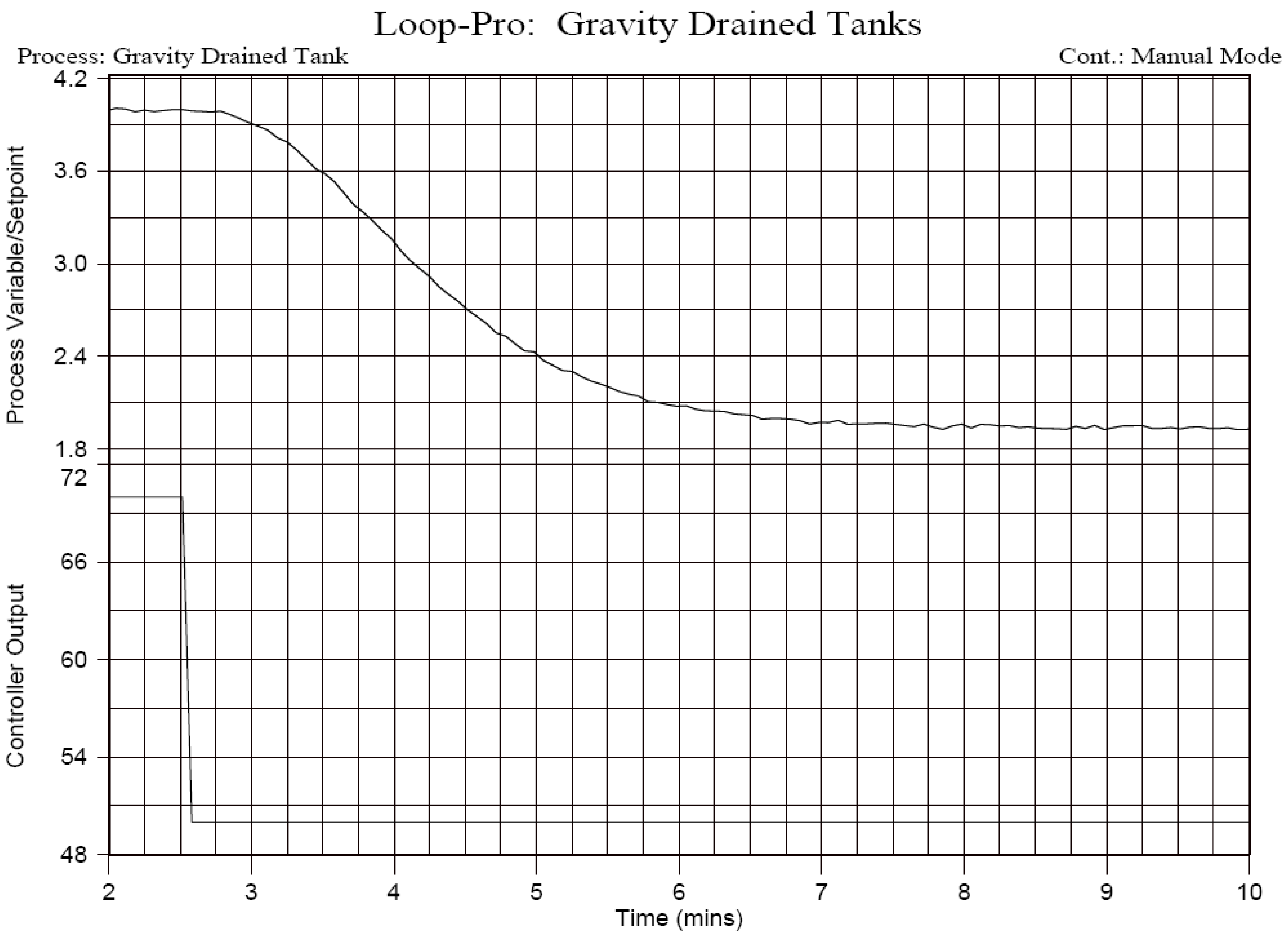
Gravity Drained Tanks is Nonlinear



A controller should be designed for a specific level of operation!

Manual Fitting (FOPDT) - Practice

1. Find $K_p = \Delta y / \Delta u$
2. Find θ_p
3. Find $y_{0.632}$
4. Find $t_{0.632}$
5. Find τ_p

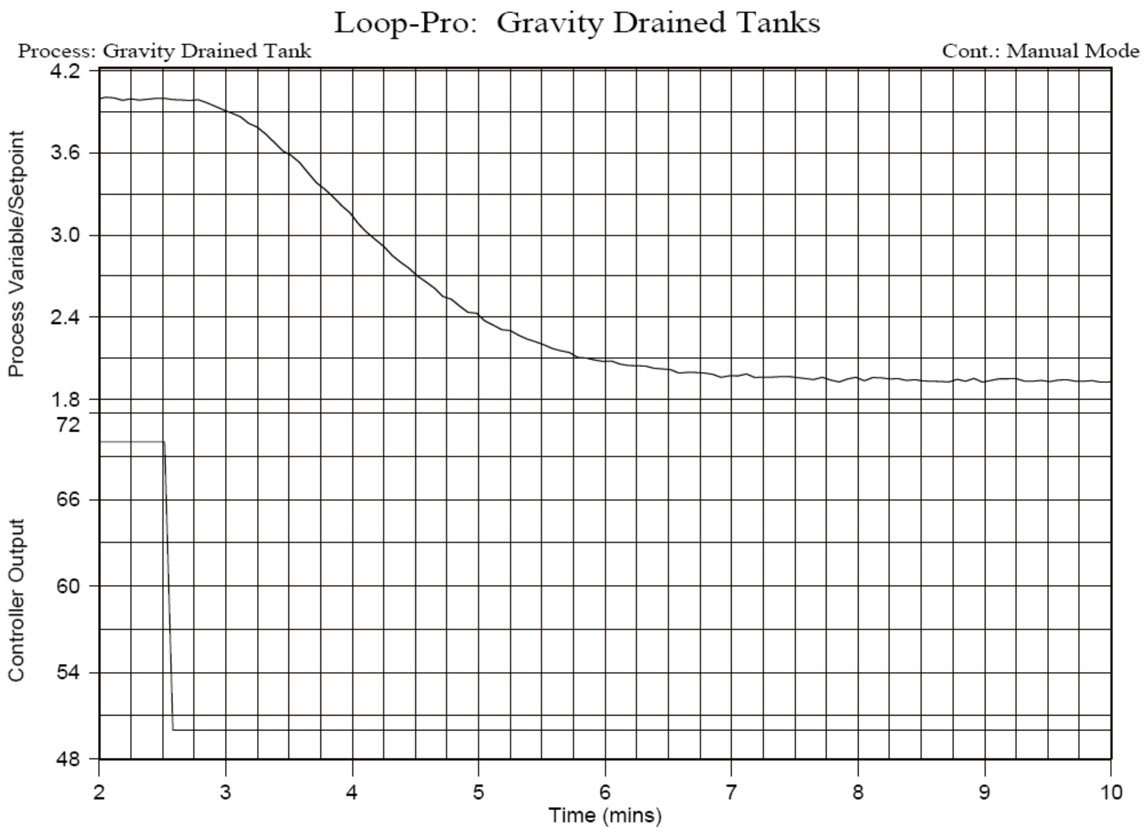


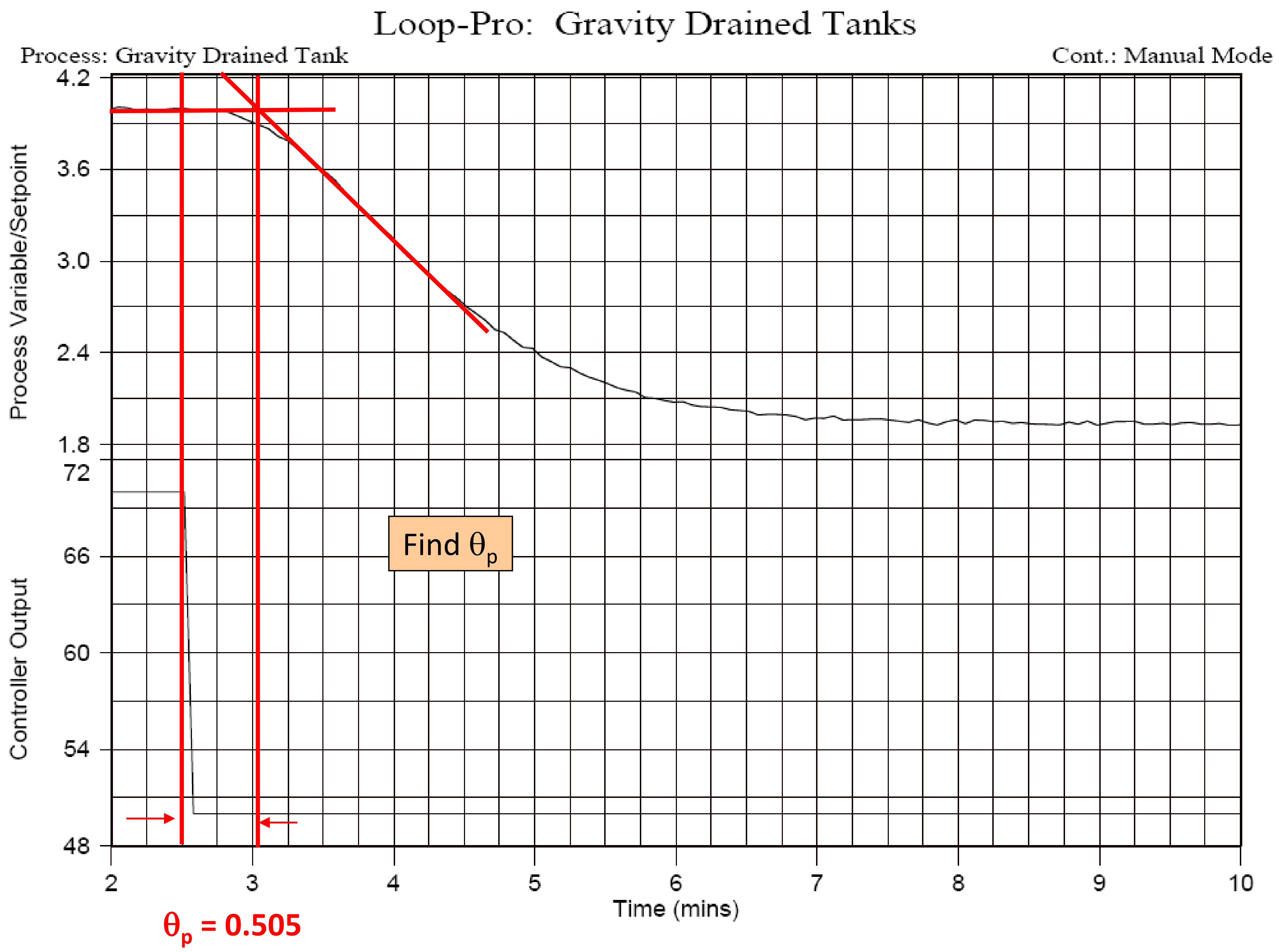
Loop-Pro: Gravity Drained Tanks
Process: Gravity Drained Tank
Cont.: Manual Mode
Process Variable/Setpoint
4.2
3.6
3.0
2.4
1.8
Controller Output
72
66
60
54
48
2
3
4
5
6
7
8
9
10
Time (mins)

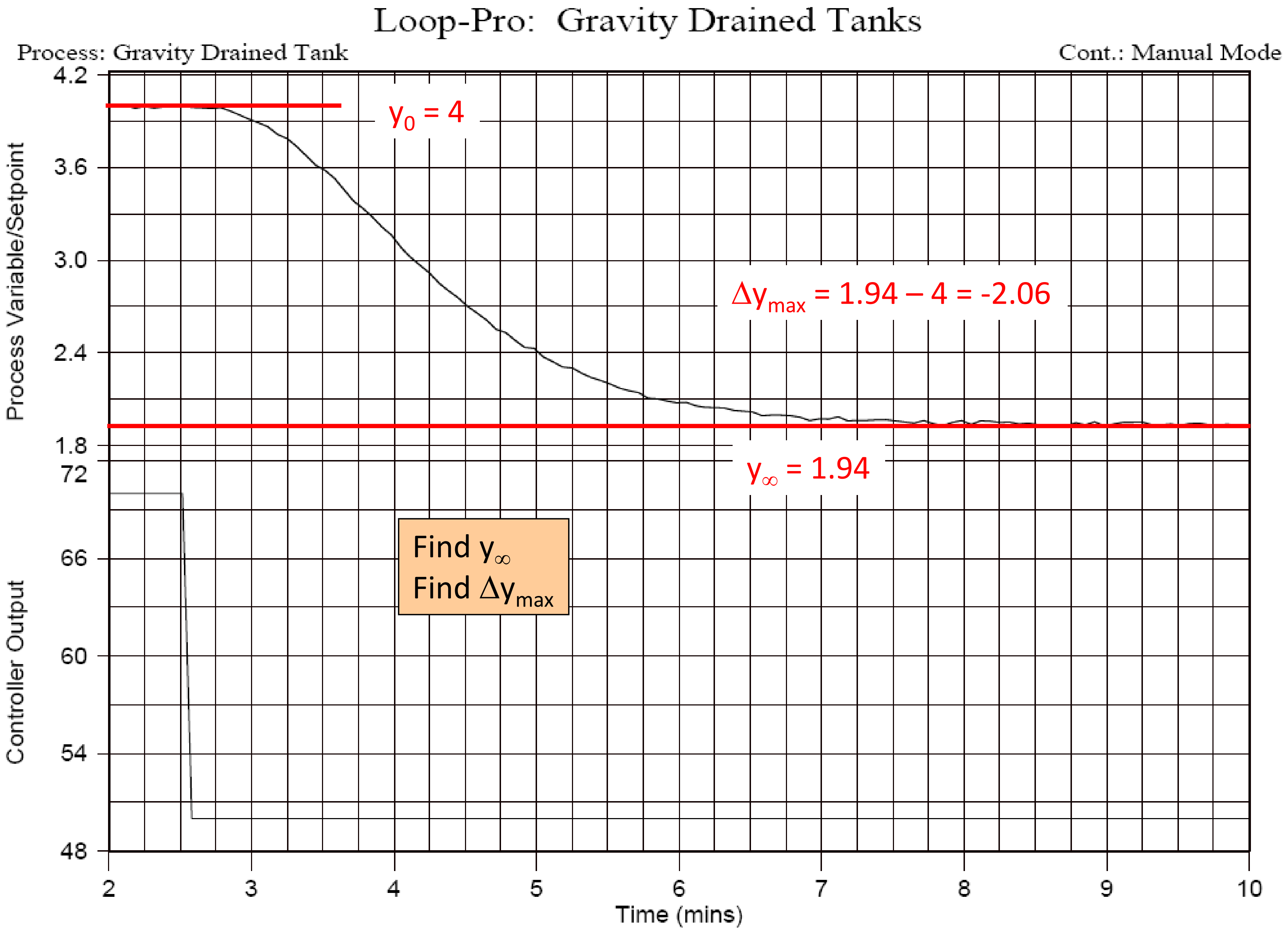
Find K_p

- $K_p = \Delta y_{max} / \Delta u = -2.06\ m/-15\%$

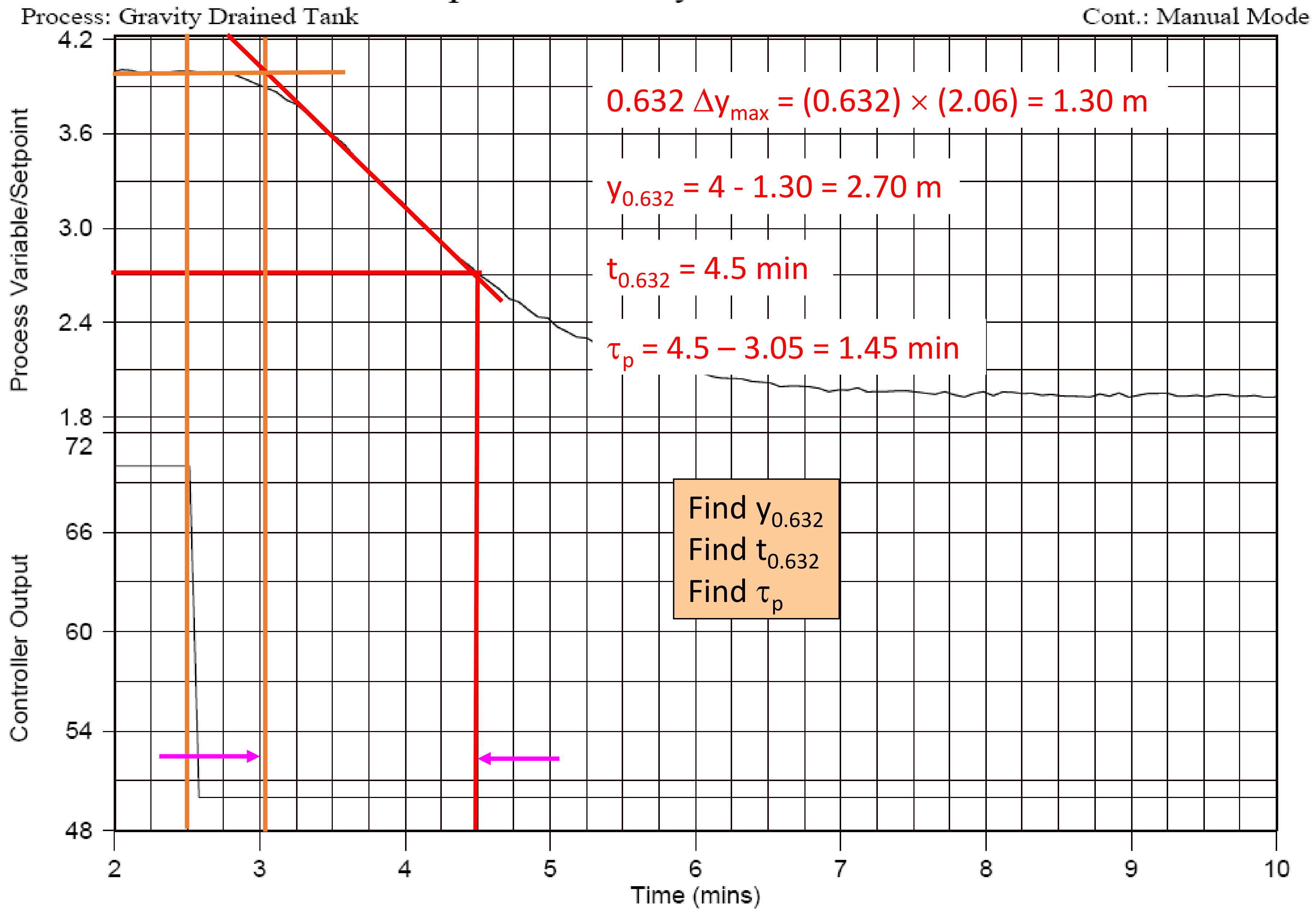
  $= 0.137\ m/\%$




Loop-Pro: Gravity Drained Tanks
Process: Gravity Drained Tank
Cont.: Manual Mode
Process Variable/Setpoint
4.2
3.6
3.0
2.4
1.8
72
66
60
54
48
Controller Output
Find θ_p
2
3
4
5
6
7
8
9
10
Time (mins)
θ_p = 0.505


Loop-Pro: Gravity Drained Tanks
Process: Gravity Drained Tank
Cont.: Manual Mode
Process Variable/Setpoint
Controller Output
4.2
3.6
3.0
2.4
1.8
72
66
60
54
48
2
3
4
5
6
7
8
9
10
Time (mins)
y0 = 4
Δymax = 1.94 – 4 = -2.06
y∞ = 1.94
Find y∞
Find Δymax

Loop-Pro: Gravity Drained Tanks



Caution: Account for dead time when calculating τ_p!