# Controller Transfer Functions

# Proportional-Integral-Derivative (PID) Control

## *PID Control*

The *parallel form* of the PID control algorithm (without a derivative filter) is given by

- Many variations of PID control are used in practice.

$$p(t) = \bar{p} + K_c \left[ e(t) + \frac{1}{\tau_I} \int_0^t e(t^*)\, dt^* + \tau_D \frac{de(t)}{dt} \right] \qquad (8\text{-}13)$$

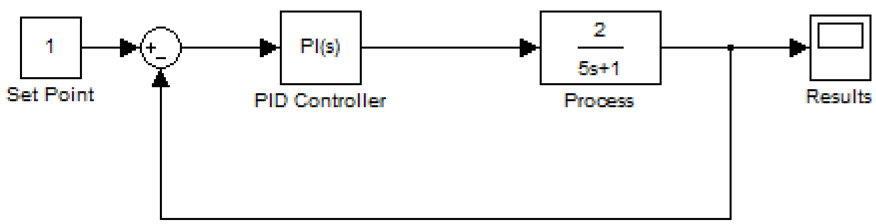
The corresponding transfer function is:

$$\frac{P'(s)}{E(s)} = K_c \left[ 1 + \frac{1}{\tau_I s} + \tau_D s \right] \qquad (8\text{-}14)$$

# Using the Controller Transfer Function

$$\frac{P'(s)}{E(s)} = K_c\left[1 + \frac{1}{\tau_I s} + \tau_D s\right] \quad (8\text{-}14)$$

# MATLAB Example (Simulink)



# System Transfer Function

$$\frac{Y(s)}{T(s)} = \frac{\tau_I s + 1}{\left(\frac{5\tau_I}{2K_c}\right)s^2 + \left(\frac{\tau_I + 2\tau_I K_c}{2K_c}\right)s + 1}$$

$$G(s) = \frac{K}{\tau^2 s^2 + 2\zeta\tau s + 1}$$

# Can we use Eqn 5-53 to specify an Overshoot?

$$OS = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$

# MathCAD Solution – 15% Overshoot

$OS := .15 \quad \tau_I := .5 \quad \zeta := 0.5 \quad \tau_P := 1.0 \quad K_c := 1$

Given

$$\tau_P = \sqrt{\frac{5 \cdot \tau_I}{2 \cdot K_c}} \qquad \zeta = \frac{1}{2\tau_P} \cdot \left( \frac{\tau_I + 2 \cdot K_c \cdot \tau_I}{2 \cdot K_c} \right) \qquad OS = \exp\left( \frac{-\pi \cdot \zeta}{\sqrt{1 - \zeta^2}} \right)$$

$$\begin{pmatrix} \tau_{I.sol} \\ \zeta_{sol} \\ \tau_{P.sol} \end{pmatrix} := \text{Find}(\tau_I, \zeta, \tau_P) \qquad \begin{pmatrix} \tau_{I.sol} \\ \zeta_{sol} \\ \tau_{P.sol} \end{pmatrix} = \begin{pmatrix} 1.188 \\ 0.517 \\ 1.723 \end{pmatrix}$$

Doesn't perfectly apply because there is also a zero - this will affect the overshoot as well.

# MATLAB Simulation – Why 20% Overshoot?