## Special Problem 12

Consider the isothermal reactor system shown below. The objective of the proportional-only control system is to maintain the concentration of species A in the reactor at a constant value. Species A reacts with B to form the desired product. Note that $A$ is considerably more expensive than $B$. The flow rate $q_{A}$ is set by a metering pump and is constant at $10 \mathrm{ft}^{3} / \mathrm{min}$; it consists of a combination of a stream of pure A and a water dilution stream. The relative amounts of these two streams, and hence the concentration of $A\left(c_{A}\right)$ entering the pump (after mixing), can be adjusted by changing the position of the control valve shown in the diagram. The relationship between $c_{A}$ and the input signal to the valve is approximately:

$$
c_{A}=4.5\left(p_{v}-3\right)^{1 / 5}
$$

where $\bar{p}_{v}$ at the expected steady-state operating conditions is 8 psi . The flow rate, $\mathrm{q}_{\mathrm{B}}$, is also constant at a rate of $5 \mathrm{ft}^{3} / \mathrm{min}$. However, the concentration of $B\left(c_{B}\right)$ varies with time and represents the principal disturbance in the system. There is a long section of pipe, 60 ft in length and 6 inches in diameter, between the metering pump and the reactor. The dynamics of the linear constant pressure drop valve are fast and can be neglected. Also, the dynamics associated with the concentration measurement can be approximated simply as a gain with 30 seconds ( 0.5 min ) of dead time. The concentration range on the transducer is set to $0-8 \mathrm{lbmoles} / \mathrm{ft}^{3}$. The transfer functions which relate $\mathrm{c}_{\mathrm{A}, \text { out }}$ to the reactor inlet concentration of $\mathrm{A}\left(\mathrm{c}_{\mathrm{A}, \mathrm{in}}\right)$ and $B\left(C_{B}\right)$ respectively are:

$$
\begin{aligned}
& \frac{C_{A, \text { out }}(s)}{C_{A, \text { in }}(s)}=\frac{0.06}{3.9 s+1} \\
& \frac{C_{A, \text { out }}(s)}{C_{B}(s)}=\frac{-0.05}{3.9 s+1}
\end{aligned}
$$

a) Add the feedback control loop to the diagram below (draw it in).
b) Draw a block diagram for the feedback control loop for this system. Derive a mathematical expression for each transfer function, including numbers and units. Label the diagram with variables and units.
c) Should the valve be air-to-open or air-to-close? Should the controller be direct acting or reverse acting? Please justify your responses.
d) Determine the open-loop transfer function $G_{\text {OL }}$. How does this compare with the transfer function you would get from an open loop step test? Assume P-only control and use $\mathrm{K}_{\mathrm{c}}$ for the transfer function of the controller.
e) For a proportional-only controller, what final value does the system reach in response to a step change in the set point? (Note: express your answer in terms of $\mathrm{K}_{\mathrm{c}}$ )


