Problem 1a

$$\min_{u(t)} x_2(t_f)$$
$$subject\ to$$
$$\frac{dx_1}{dt} = u$$
$$\frac{dx_2}{dt} = x_1^2 + u^2$$
$$x(0) = [1\ 0]^T$$
$$t_f = 1$$

Problem 1b

$$\min_{u(t)} x_2(t_f)$$
$$subject\ to$$
$$\frac{dx_1}{dt} = u$$
$$\frac{dx_2}{dt} = x_1^2 + u^2$$
$$x(0) = [1\ 0]^T$$
$$x_1(t_f) = 1$$
$$t_f = 1$$

Problem 2

$$\min_{u(t)} x_4(t_f)$$
$$subject\ to$$
$$\frac{dx_1}{dt} = x_2$$
$$\frac{dx_2}{dt} = -x_3 u + 16t - 8$$
$$\frac{dx_3}{dt} = u$$
$$\frac{dx_4}{dt} = x_1^2 + x_2^2 + 0.0005(x_2 + 16t - 8 - 0.1x_3 u^2)^2$$
$$x(0) = [0\quad -1\quad -\sqrt{5}\quad 0]^T$$
$$-4 \le u \le 10$$
$$t_f = 1$$

Problem 3

$$\max_{u(t)} \; x_2(t_f)$$

$$subject\ to$$

$$\frac{dx_1}{dt} = -(u + 0.5u^2)x_1$$

$$\frac{dx_2}{dt} = u\, x_1$$

$$x(0) = [1\ 0]^T$$

$$0 \le u \le 5$$

$$t_f = 1$$

Problem 4

$$\max_{T(t)} \; x_2(t_f)$$

$$subject\ to$$

$$\frac{dx_1}{dt} = -k_1 x_1^2$$

$$\frac{dx_2}{dt} = k_1 x_1^2 - k_2 x_2$$

$$k_1 = 4000 \exp\left(-\frac{2500}{T}\right)$$

$$k_2 = 6.2e5 \exp\left(-\frac{5000}{T}\right)$$

$$x(0) = [1\ 0]^T$$

$$298 \le T \le 398$$

$$t_f = 1$$

Problem 5

$$\max_{u(t)} \; 1 - x_1(t_f) - x_2(t_f)$$

$$subject\ to$$

$$\frac{dx_1}{dt} = u\,(10\,x_2 - x_1)$$

$$\frac{dx_2}{dt} = -u\,(10\,x_2 - x_1) - (1 - u)\,x_2$$

$$x(0) = [1\ 0]^T$$

$$0 \le u \le 1$$

$$t_f = 12$$