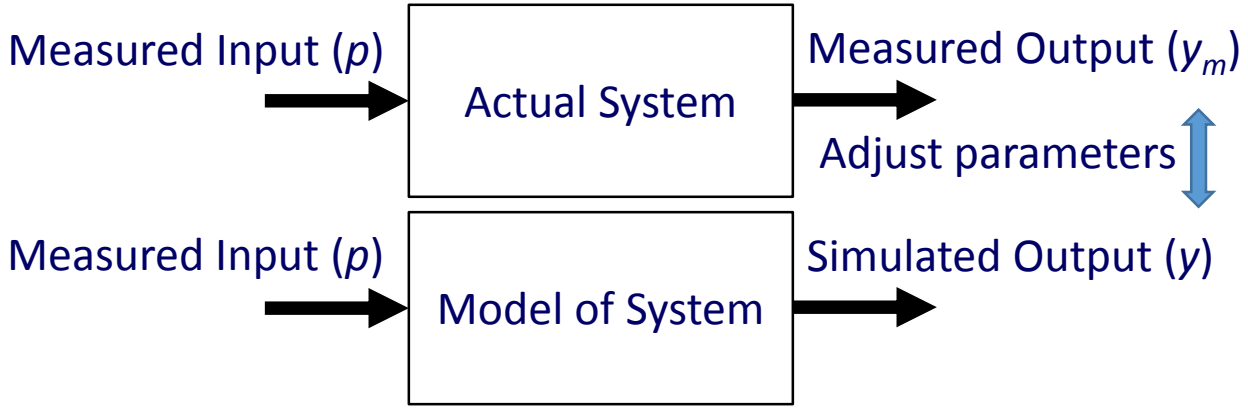
Part II: Dynamic Estimation

- Dynamic Modeling
  - Empirical
  - Fundamental
- **Dynamic Estimation**
- Dynamic Control



$$\begin{aligned} \text{minimize} \quad & \|y_m - y\|_n \\ subject\ to \quad & 0 = f\left(\frac{dx}{dt}, x, y, p\right) \\ & 0 \leq g\left(\frac{dx}{dt}, x, y, p\right) \end{aligned}$$

Data for Empirical Modeling

- Linear first-order equation
  - $\tau \frac{dv}{dt} = -v + K\ p$
- Gain (K)
  - Change in output / Change in input with $K = \frac{\Delta v}{\Delta p} = \frac{v_2 - v_1}{p_2 - p_1}$
  - Points 1 and 2 at steady state, $\frac{dv}{dt} = 0 = -v + K\ p$ or $v = K\ p$
    - Point 1: $p_1 = 0, v_1 = 0$ (at rest)
    - Point 2: $p_1 = 20, v_1 = 0$
- Time constant (τ)
  - Step change in the input
  - Time to reach 63.2% of steady state value

Why is τ the time for 63.2% to steady-state?

Separate and Integrate

$$\tau \frac{dv}{dt} = -v + K\,p$$

$$\int_0^v \frac{\tau}{Kp - v} dv = \int_0^t dt$$

$$-\tau \ln\left(\frac{Kp - v}{Kp}\right) = t$$

$$v(t) = Kp\left(1 - \mathrm{e}^{-t/\tau}\right)$$

$v(\tau) = Kp\left(1 - \mathrm{e}^{-\tau/\tau}\right)$ with $t=\tau$

$$v(\tau) = Kp\,(0.632)$$

$v(\infty) = Kp$ at steady state

Laplace Transforms

$$\tau \frac{dv}{dt} = -v + K\,p$$

$$\tau s V(s) - v(0) = -V(s) + KP(s)$$

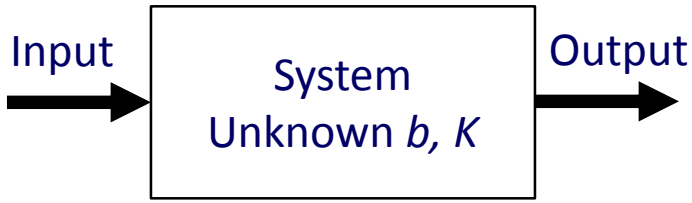
$$\boxed{\frac{V(s)}{P(s)} = \frac{K}{\tau s + 1}}$$ Transfer Function

$$V(s) = \frac{1}{s}\left(\frac{K\,p}{\tau s + 1}\right)$$ P(s) = p/s (step function)

$$v(t) = Kp\left(1 - e^{-\frac{t}{\tau}}\right)$$ Inverse Laplace Transform

$$v(\tau) = Kp\left(1 - e^{-1}\right) = Kp(0.632)$$ At $t = \tau$

Dynamic Estimation in Excel

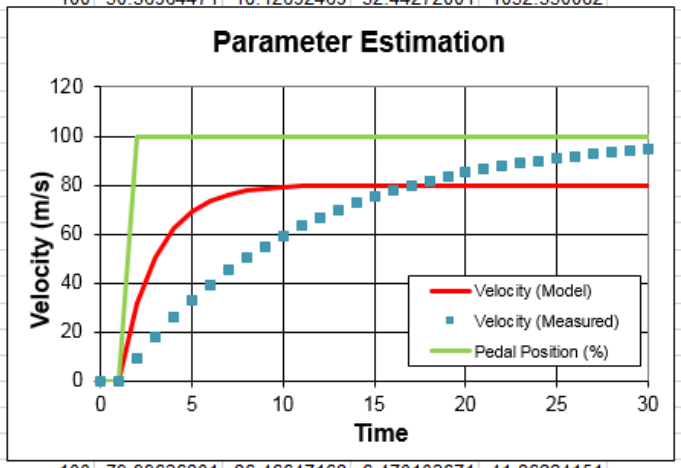


$$\underset{K,\tau}{\text{minimize}} \quad (v_m - v)^2$$

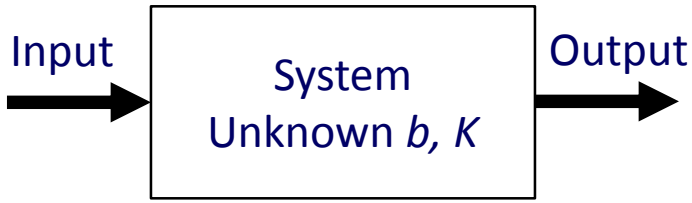
$$s.t. \quad \tau \frac{dv}{dt} = -v + K\,p$$

$$0 \le b \le 100$$

	A	B	C	D	E	F	G	H	I	J	K
1	**Parameter Estimation**				Change	u	y	Target			
2				time	Delta MV	manipulated	model	desired	abs(error)	error^2	
3				0		0	0	0	0	0	
4				1	0	0	0	0	0	0	
5	**Model Parameters**			2	100	100	31.47754722	9.516258196	21.96128903	482.2982157	
6	**gain (K)**	0.8		3	0	100	50.56964471	18.12692469	32.44272001	1052.530082	
7	**tau (τ)**	2		4	0						
8				5	0						
9	**Measured Velocity**			6	0						
10	**Final Target**	100		7	0						
11	**Time Constant (tau)**	10		8	0						
12	**Delay (theta)**	1		9	0						
13				10	0						
14				11	0						
15	**Minimize Either of These**			12	0						
16	**Sum of Squared Errors**	9884.722		13	0						
17	**Sum of Absolute Errors**	435.9744		14	0						
18				15	0						
19				16	0						
20				17	0						
21				18	0						
22				19	0						
23				20	0						
24				21	0	100	79.99636801	86.46647168	6.470103671	41.86224151	



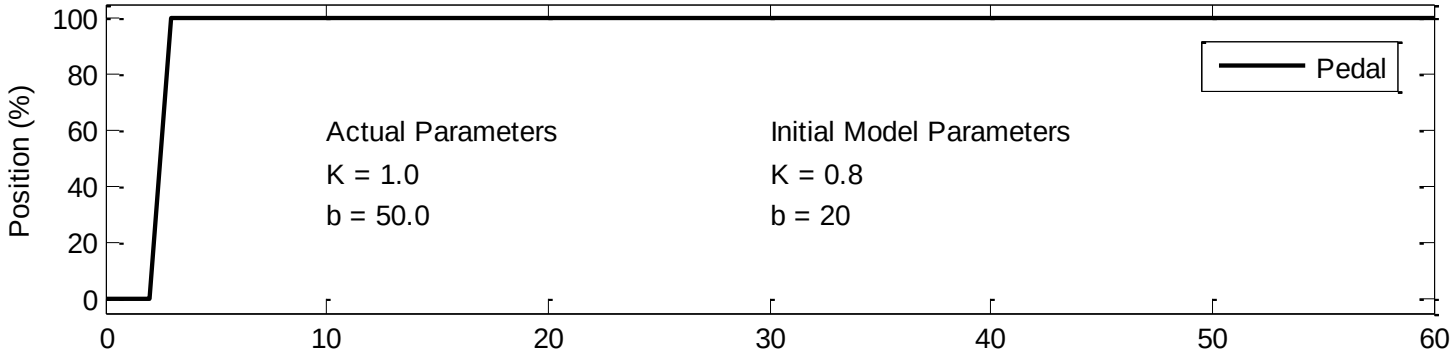
Dynamic Estimation in MATLAB

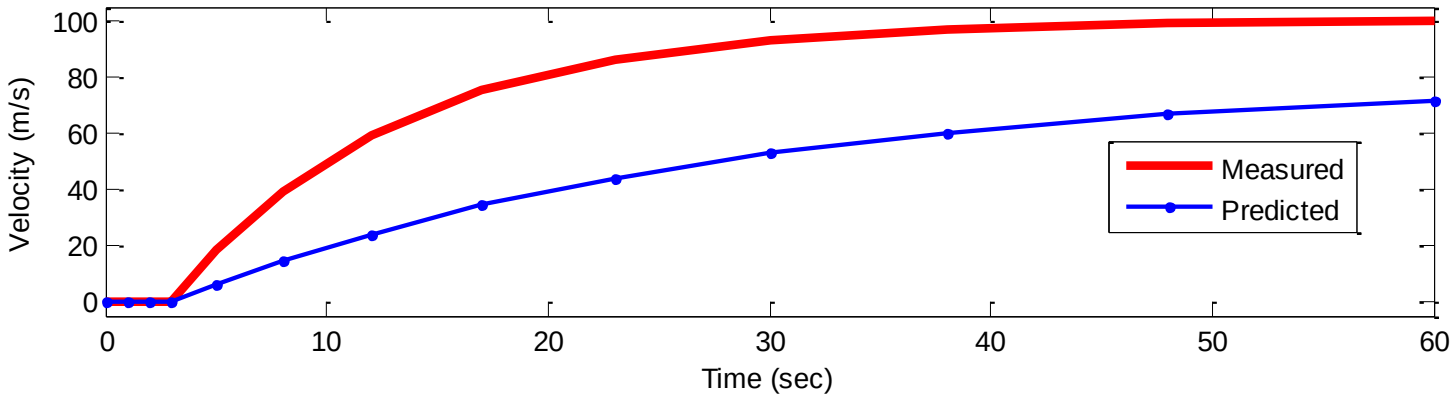


$$\underset{K,b}{\text{minimize}} \quad (v_m - v)^2$$

$$s.t. \quad \frac{m}{b}\frac{dv}{dt} = -v + K\,p$$

$$0 \le b \le 100$$





Dynamic Estimation in MATLAB

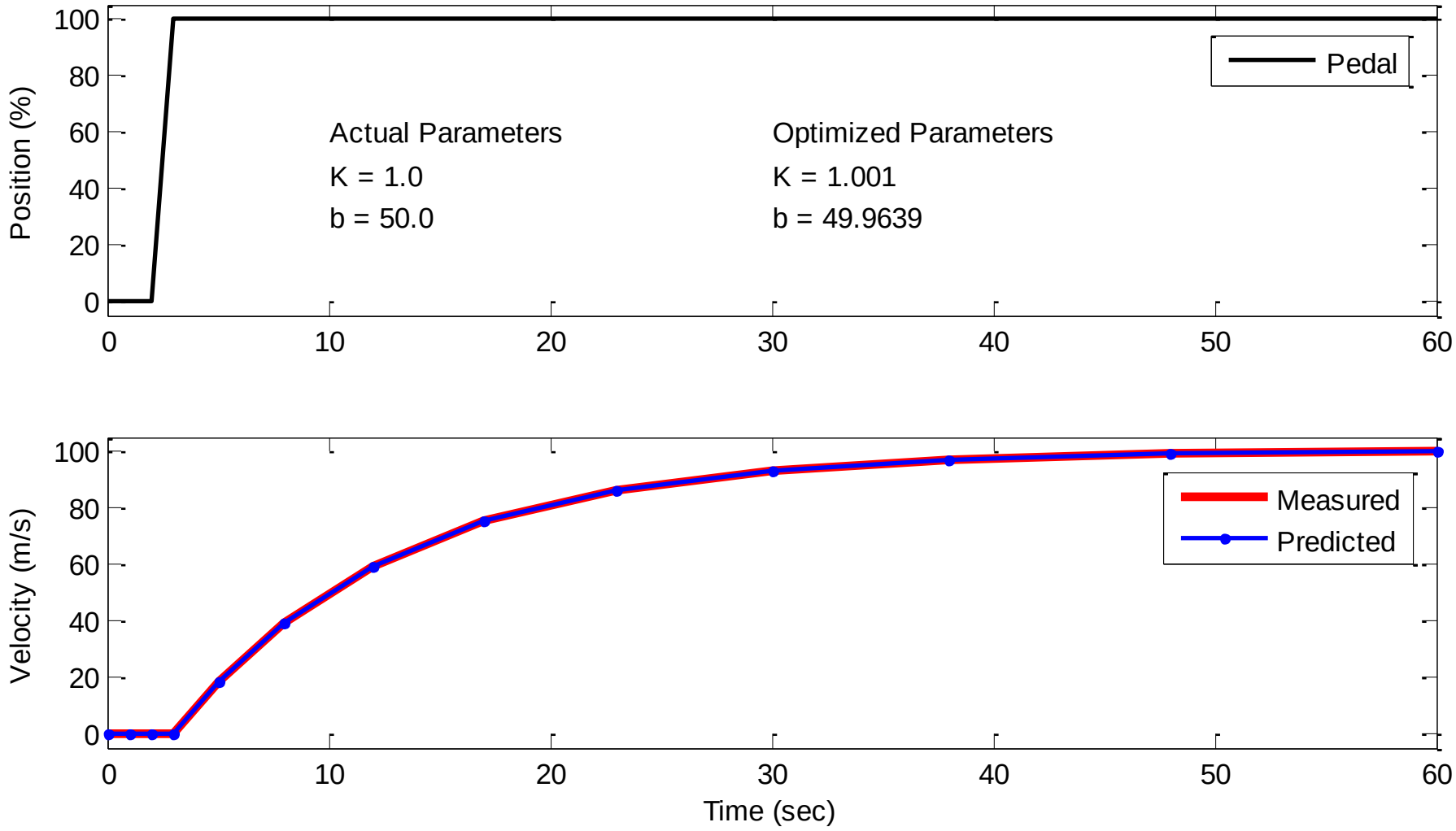
Setup

```
clear all; close all; clc % clear session
addpath('apm') % load APMonitor.com  toolkit

% s = server, a = application
s = 'http://byu.apmonitor.com';
a = 'velocity';

% clear prior application
apm(s,a,'clear all');

% load model and data
apm_load(s,a,'ferrari.apm');
csv_load(s,a,'ferrari.csv');

% specify parameters
apm_info(s,a,'FV','K');
apm_info(s,a,'FV','b');
apm_info(s,a,'CV','v');
```

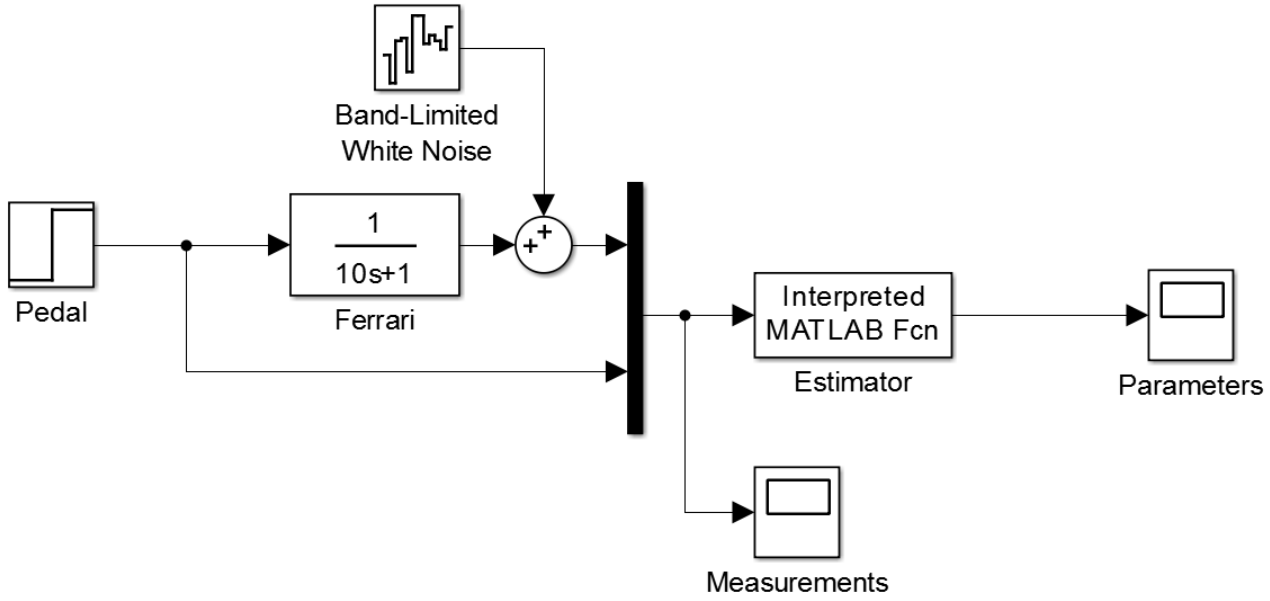
Tune and Solve

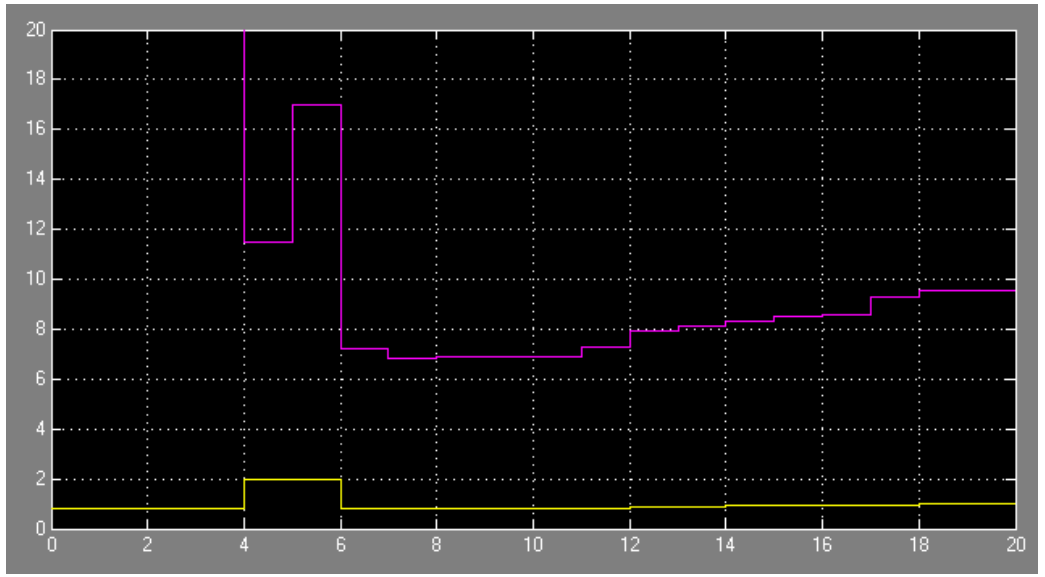
```
% configuration parameters
apm_option(s,a,'nlc.imode',5);
apm_option(s,a,'nlc.nodes',3);

% solve and retrieve results
output = apm(s,a,'solve'); disp(output);
y = apm_sol(s,a); z = y.x;

% open web-viewer
apm_web(s,a);
```

Dynamic Estimation MATLAB Results



Dynamic Estimation in Simulink





Dynamic Estimation Contour Plot