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Dr. Hedengren:

Compressors in gas pipelines are designed to maintain pressure and flow despite many flow disturbances that can occur during transport. The most common control implementation for gas compressors utilizes a recycle bleed stream that essentially recycles a portion of the pressurized stream in order to maintain a pressure or flow set point. This process wastes energy and shows a slow response time because of valve dynamics. A. Cortinovis et.al. developed a linear MPC controller to accomplish the same control objective but instead manipulates the compressor driver torque in order to control the discharge pressure and flow rate. Their study showed that controller settling time decreased by about 50% using the MPC.

Because gas pipelines utilize many compressors along the length of the supply line, it is desirable to develop a reliable, energy efficient control scheme that can achieve anti-surge and process control for gas compressors in series. This work explores a linear MPC to control a system of two compressors in series. We extend the model developed by A. Cortinovis et.al., simulate the model in Simulink, and extract a linear state-space model for use in the linear MPC. The linear MPC maintains a pressure set point in each compressor, while manipulating driver torque to achieve the set point. Currently, the MPC works in tandem with a separate PI controller, which acts as the recycle anti-surge control utilized in current practices. The set point tracking and disturbance rejection ability of the controller are then tested to show the controller performance.

This linear MPC is a first-step towards implementing this controller in a physical plant. Future work will combine the anti-surge control together with the pressure tracking MPC in a non-linear MPC. This will allow for a more robust controller that can then be tested on an experimental pilot system.

Sincerely,

Aaron Bush

Brandon Hillyard

Highlights:

- Linear MPC for a gas compressor system is extended for use with two gas compressors in series
- A first-principles model of the two compressor system is simulated in Simulink
- A state-space model is extracted from the full model and implemented in a linear MPC in tandem with a recycle valve PI controller
- The controller responds quickly to disturbances and set point changes, achieving the desired anti-surge control
- Competition between the two separate controllers causes unexpected oscillations and loss of control; future work will combine both controllers into one nonlinear model predictive control scheme

LINEAR MODEL PREDICTIVE CONTROL AND ANTI-SURGE CONTROL FOR CENTRIFUGAL GAS COMPRESSORS IN SERIES

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LINEAR MODEL PREDICTIVE CONTROL AND ANTI-SURGE CONTROL FOR CENTRIFUGAL GAS COMPRESSORS IN SERIES

Introduction

Compressors in gas pipelines are designed to maintain pressure and flow despite many flow disturbances that can occur during transport. Conventional control schemes employed in gas compressor systems include a recycle "bleed" stream that maintains a pressure set point by recycling gas. This set up is an effective means of eliminating compressor surge, a condition of flow instability that arises when the ratio of downstream to upstream pressure exceeds an amount unacceptable for the flow rate. However, because the recycle stream essentially wastes energy and money used to pressurize gas in the first place, this set up is not an ideal control method for anti-surge, especially in a situation where two compressors are linked in series.

One method developed in a work by A. Cortinovis et.al. explores the use of linear model predictive control (MPC) to manipulate the driver torque of a compressor linked to a variable speed drive (VSD) in order to reduce the speed of the compressor and thus more effectively control the compressor output. The results of that study show that the VSD responds more rapidly than a recycle control valve, and allows for a more efficient compressor set up.

The purpose of this work is to extend the first-principles model developed by A. Cortinovis et.al. for two compressors in series. The model is simulated using step responses to show the behavior of the system. A linear state space model is then extracted for use with a linear MPC which manipulates the driver torque of the two compressors to maintain discharge pressure set points for the system. The MPC is implemented in tandem with a standard anti-surge recycle controller as a first step in the controller design to improve the controller performance. The performance of the controller is then discussed, and future work is laid forth.

Literature Review

The work performed by A. Cortinovis et.al. develops an improved method for controlling gas compressors and eliminating surge than the current industry practice. This method relies on a linear model predictive controller for anti-surge and process control, similar to the method explored in this work. A first principles model relating pressures, flow rates, and valve positions serves as the basis for the controller model, which is linearized and discretized at each time step before implementation in the solver. The model dynamic and static parameters are validated using an experimental test rig, after which the controller performance is compared with a traditional PI recycle controller. The MPC compared to the PI controller reduces settling time by 50%, and the distance to surge by up to 11%, exhibiting the value of the MPC controller for this application.

This study builds upon the work established by A. Cortinovis et.al. by applying the linear MPC to two compressors in series. The same parameters developed in the single compressor model are extended for use in the two compressor model. Because the nonlinearities that exist in the single compressor system are multiplied when two compressors are coupled together, this work will explore whether a linear MPC set up is a desirable control scheme. The MPC is tested in order to determine the set point tracking and disturbance rejection ability of the system. Failure of the controller in these tests may indicate that a nonlinear MPC is a more favorable solution to the serial compressor model.

Theory

Figure 1 graphically represents the operating points of a gas compressor. The vertical axis represents the pressure ratio (Π) of the compressor, while the horizontal axis represents the mass flow rate of the compressor q_c. Optimal operation occurs along a line of peak efficiency (represented by curved lines) while optimizing the distance (SD) from the surge line (SL) and the choke line (CL). Crossing SL can lead to serious damage in the compressor and reverse flow conditions. To tighten control further and remove the possibility of crossing the SL, a surge control line (SCL) may be set a specified distance from the SL. It is desired to maintain operation an optimal distance from SCL and CL to avoid surging and deliver flow at the most efficient conditions.

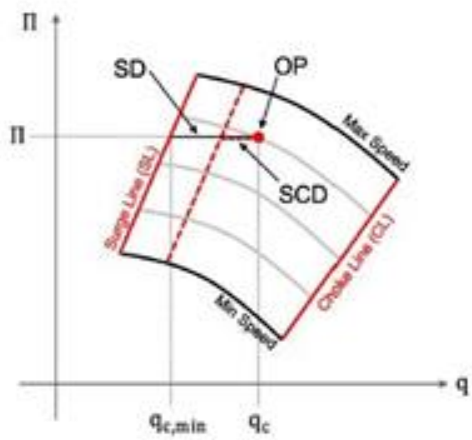

FIGURE 1 - COMPRESSOR MAP SHOWING THE OPTIMAL OPERATING POINT

Methods

This section details the first principle equations upon which a simulation of the model is built in Simulink. The equations and parameters used in this model are identical to those used in A. Cortinovis et.al., and are simply duplicated for the second compressor and linked together with an intermediate tank to decouple the systems. The process is described, and the equations used to model the system in Simulink are set forth.

Process flow diagram

Figure 2 shows a schematic of the two compressors in series linked together with an intermediate tank. Air flow to the suction side of compressor 1 is regulated by a control valve with flow rate q_s, as well as a suction tank, V_s. The compressor operates with a rotational speed ω and compresses the gas with a flow rate from the compressor of q_c. The gas enters a discharge tank V_d at a pressure of p_d. An intermediate tank with pressure P_i connects the two compressors. Two control valves regulate the flow in and out of this tank, and subsequently the flow into the next compressor.

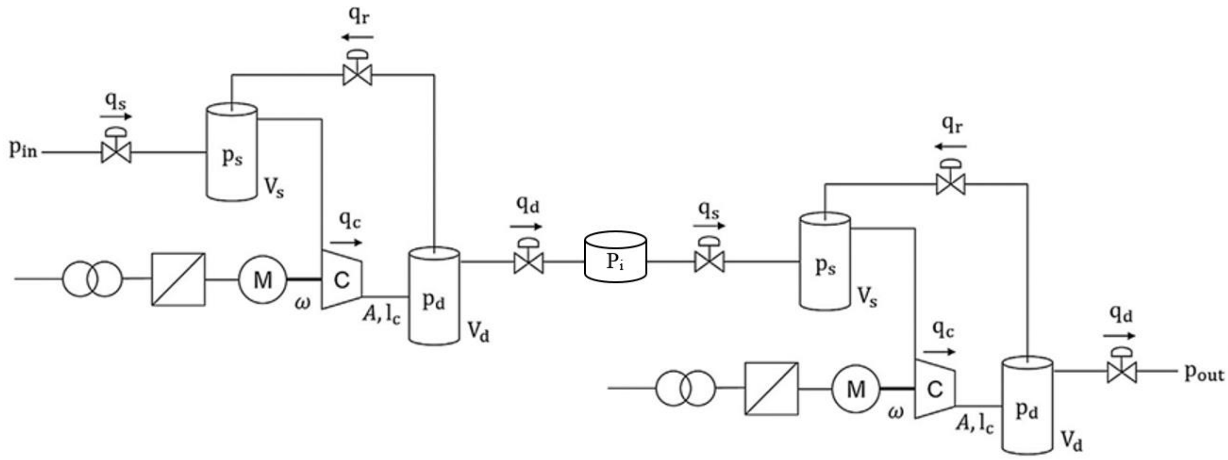

FIGURE 2 - PROCESS FLOW DIAGRAM OF TWO GAS COMPRESSORS IN SERIES WITH RECYCLE AND AN INTERMEDIATE TANK

Inputs and Outputs

The system as shown in Figure 2 is modeled in Simulink in order to characterize the input to output relationships of the model. Each compressor system takes the suction valve openings, recycle rates, and motor torque speeds of each compressor as inputs to the system, and outputs a pressure and compressor mass flow rate. The intermediate tank P_i serves as a buffer between the two compressor systems to effectively decouple the compressors and maximize control of the system. The output pressure of the intermediate tank is based on the mass flow out of the first compressor (q_d) and mass flow into the second compressor (q_s). The size of the intermediate tank determines how quickly changes will occur to one compressor based on changes in operation from the other compressor. For the purposes of this exercise, the volume of this tank is chosen to be the same as the suction and discharge tanks shown in Figure 2.

Model Equations

The system is modeled using dynamic equations to represent the change in pressure as a function of mass flow rate across the compressors. The equations are equivalent to those presented in the work by A. Cortinovis et.al. and are listed for reference. The gas flow rates are functions of the valve position (u), pressure (p), and ambient pressure (p_a), and are based on the simplified Bernoulli throttle equation.

$$q_s = q_s(u_{in}, p_s, p_a) \tag{1}$$

$$q_r = q_r(u_r, p_s, p_d) \tag{2}$$

$$q_d = q_d(u_{out}, p_a, p_d) \tag{3}$$

The dynamic pressure relationships are given by the following equations, derived from a mass balance on the suction, discharge, and intermediate tank (where a is the speed of sound and V is the volume of the tank):

$$\frac{dp_s}{dt} = \frac{a^2}{V_s}(q_s + q_r + q_c) \tag{4}$$

$$\frac{dp_d}{dt} = \frac{a^2}{V_d}(q_c - q_r - q_d) \quad (5)$$

$$\frac{dp_{int}}{dt} = \frac{a^2}{V_{int}}(q_{d1} - q_{s2}) \quad (6)$$

A mass balance on the compressor yields the following relationship for the compressor flow rate,

$$\frac{dq_c}{dt} = \frac{A}{l_c}(\Pi^{ss}(\vec{\alpha}, \omega, q_c)p_s - p_d) \quad (7)$$

where A is the piping cross section, l_c the duct length, and Π^{ss} is a fitted polynomial map for the steady state pressure ratio.[1] Finally, an equation relating the variation of the compressor speed with respect to torque is shown in Equation 8, where J is the inertia of the system, τ_d is the driver torque input to the system, τ_c is the torque from the air compression fitted to a steady state map with parameters $\vec{\beta}$, as described in detail in A. Cortinovis et.al.

$$\frac{d\omega}{dt} = \frac{1}{J}\left(\tau_d - T_c\left(\vec{\beta}, \omega, q_c\right)\right) \quad (8)$$

Simulation Results

A model of two compressors in series is created utilizing the single compressor first principles model developed by A. Cortinovis et.al. This compressor series model is implemented using Simulink in Matlab 2015b. In order to develop a linear state space model it is necessary to perform step tests from steady state operation. Steady state operation is determined by the intermediate tank pressure. In order to achieve steady state, torque of compressor 1 is adjusted until a constant pressure is observed in the intermediate tank. Step tests are then performed by changing torque on each compressor, and the response in outlet pressure and flow is recorded. The step value and step response are then used in developing a linear state space model for use in the MPC.

Linear Model for MPC

The steady-state first principles is then linearized to a form compatible with the linear MPC. The inputs of this linear model are the driver torques of compressor 1 and 2 respectively, and the outputs are the mass flow rates and discharge pressures of each compressor, as well as the pressure of the intermediate tank. The linear model extracted from the system consists of 2nd-4th order state space models. The full models are shown in the Appendix. The two compressors are coupled, despite the presence of the intermediate tank, so each output variable is affected by both the torque of compressor 1 and compressor 2.

Model Predictive Controller

The model predictive controller implemented in this process utilizes the linear state space model, as well as the full process model developed previously to perform the anti-surge and discharge pressure control. Included in the process model is the anti-surge recycle controller that utilizes the recycle line to assist with the anti-surge control.

Objective Function

The anti-surge controller is a replicate of the controller produced in the work by A. Cortinovis et.al. The same controller is implemented for both compressors, and is designed to work in tandem with the linear MPC to control surge in the system. This controller manipulates recycle valve position in order to control the location of the operating point and maintain an optimal distance from the surge line.

The objective of the linear MPC is to manipulate torque on each of the compressor drives to maintain discharge pressure set points. This is accomplished using an L1-norm objective function using the form described below in Figure 3. This objective allows the user to specify a trajectory for the controlled variable, as well as a high and low set point to bound the targeted controller response. The slack variable formulation of the error is desirable as it formulates the error as soft constraints, and thus enables a smoother controller action.

$$
\begin{aligned}
&min\phi = \mathrm{w}_{\mathrm{hi}}^{T} e_{hi} + w_{lo}^{T} e_{lo} + y^{T} c_{y} + u^{T} c_{u} + \Delta u^{T} c_{\Delta u} \\
&min\phi = \mathrm{w}_{\mathrm{hi}}^{T} e_{hi} + w_{lo}^{T} e_{lo} + y^{T} c_{y} + u^{T} c_{u} + \Delta u^{T} c_{\Delta u} \\
&\ x, y, u \\
&\text{s.t. } 0 = f\left(\frac{dx}{dt}, x, y, p, d, u\right) \\
&0 = g(x, y, p, d, u) \\
&0 \leq h(x, y, p, d, u) \\
&\tau_c \frac{dy_{t,hi}}{dt} + y_{t,hi} = sp_{hi} \\
&\tau_c \frac{dy_{t,lo}}{dt} + y_{t,lo} = sp_{lo} \\
&e_{hi} \geq y - y_{t,hi} \\
&e_{lo} \geq y_{t,lo} - y
\end{aligned}
$$

FIGURE 3 L1-NORM OBJECTIVE FUNCTION

MPC Configuration

The linear MPC uses the APOPT solver in APMonitor to solve the objective function, and uses bias updating from the full process model to update the model at each step in the time horizon. A predictive time horizon of 5 seconds ensures that the controller fully experiences the dynamics of the system and incorporates that into the control actions. The full code for the MPC controller and the linear state-space model is shown in the Appendix.

Several tuning constraints are implemented in the MPC in order to improve control and reflect the limits of the system. To improve controllability, an error band of +/-100 Pa is set around the set point, and is coded as a trajectory that re-centers after each controller cycle. Allowing the set point trajectory to re-center gives the controller greater freedom when disturbances occur or when the set point is adjusted during operation. A physical constraint on the system is the amount the torque can change in each cycle period. The torque can experience a maximum deviation of +/- 0.1 units per controller cycle (50 ms), and is coded in as a hard constraint to the controller. These tuning parameters help to make the controller more robust and able to reach the control objective.

Dynamic Optimization Results and Discussion

This section shows the results of the linear MPC implemented with bias updating from the first-principles model of the two compressors in series. The results show how pressure is able to be controlled by the linear MPC, and that surge is minimized through the performance of the anti-surge controller. A discussion on the disturbance rejection and set-point tracking ability of the controller is included. And finally, a sensitivity analysis is performed to show the steady-state and dynamic relationships between the manipulated variables and the controlled variables.

Set Point tracking

In order to determine the reliability and capability of the controller, the outlet pressure set point was adjusted for each compressor. Figure 5 shows the response for compressor 1 to set point changes, and Figure 4 shows the response of compressor 2 to set point changes. These set point changes are only on

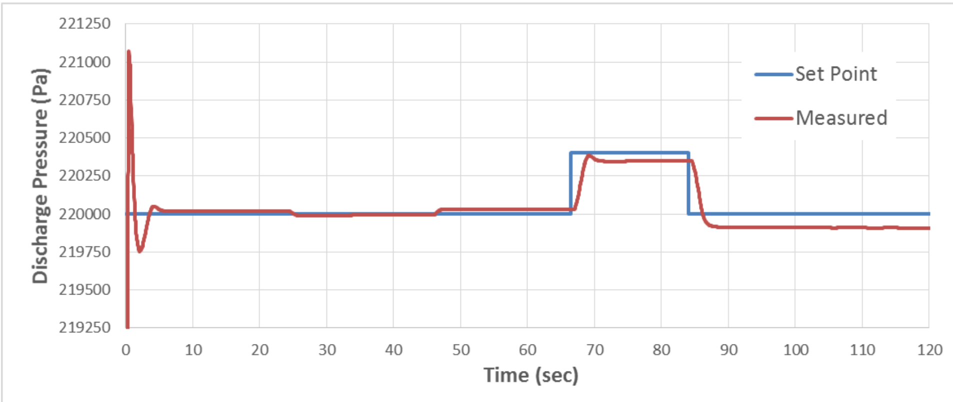

FIGURE 5 PRESSURE SET POINT RESPONSE OF COMPRESSOR 2

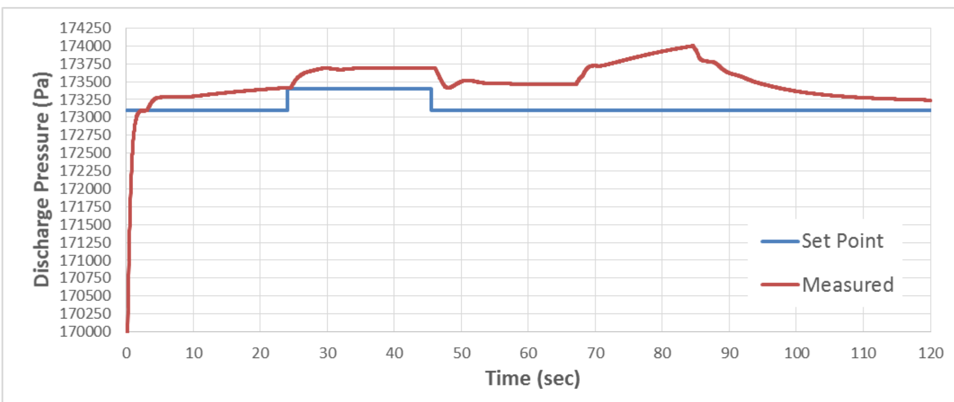

FIGURE 4 PRESSURE SET POINT RESPONSE OF COMPRESSOR 1

the scale of several hundred Pascal. As shown in Figure 5, compressor 1 suffers from some offset and drifts considerably from the set point after changes have been made. This is due to the internal response of the system anti-surge controller. The anti-surge controller and the MPC have competing effects, given time the system eventually settles back to the set-point. Compressor 2 is able to follow changes in the set-point with minor offset. For compressor 2 the anti-surge controller was disabled to better show the accuracy of the controller without the competing effects.

Disturbance Rejection

The system is also analyzed to determine the set point tracking ability of the controller. Disturbances are introduced in the form of outlet valve changes for both of the compressor systems. The outflow valve for compressor 1 was constricted from 0.45 to 0.29 after 10 seconds, and the outflow valve for compressor 2 was constricted from 0.45 to 0.35 after 40 seconds. The results of the controller performance are shown below in Figure 6 and Figure 7. The controller functions really well for the first 40 seconds of simulation time. It handles the first disturbance well, although some offset in the outlet pressure of compressor 1 remains. After the outflow is constricted in compressor 2, the controller gradually brings the discharge pressure of compressor 2 back under control by decreasing the torque of both compressors. By the time discharge pressure 2 is under control, the pressure of compressor 1 has drifted far below the set point, and is unable to recover.

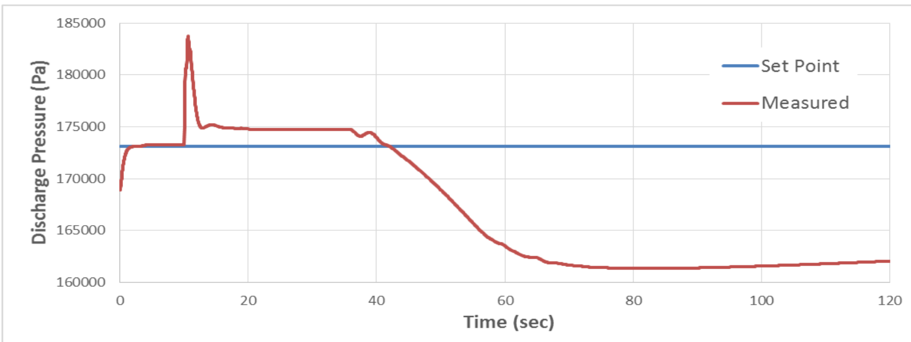

FIGURE 6 DISTURBANCE REJECTION TEST - DISCHARGE PRESSURE OF COMPRESSOR 1

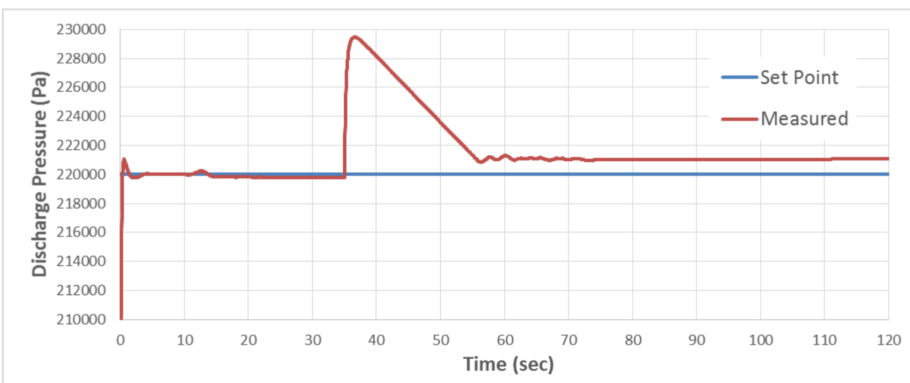

FIGURE 7 DISTURBANCE REJECTION TEST - DISCHARGE PRESSURE OF COMPRESSOR 2

Despite this, the controller seems to perform with little oscillation and with a very rapid response time, much like the MPC developed in the single compressor plant by A. Cortinovis et.al. In addition, further tests show that given more time, the pressure of compressor 1 is actually able to slowly recover back to the set point.

The unexpected performance of the controller after the 40 second mark is partially explained by the fact that changing the valve position moves the system outside the operating region that the linear MPC is designed for. Another possible explanation is that there is some competition between the anti-surge recycle controller and the MPC. Because the anti-surge controller is a Simulink based model, and the MPC is written in APMonitor using a linearized form of the model, there’s no easy way to prioritize the two control objectives, and the competing objectives drive the system away from the pressure set points. This problem can be addressed by adapting the first principles Simulink model to a form that works in APMonitor, which would allow the controller to be designed as a nonlinear MPC, and would also streamline the simulation and bias updating of the controller.

Anti-surge control

Both the set-point tracking and disturbance rejection tests show that the anti-surge objectives are able to be met. The figures below show the compressor maps, with the leftmost red line representing the surge line, the dotted line the best operating line, and the gold line the actual path that the compressor

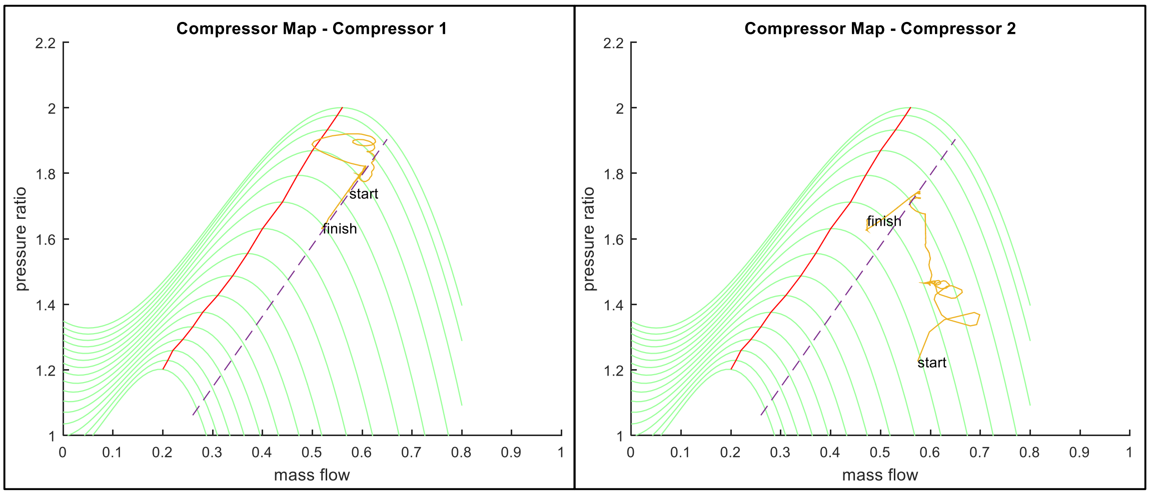

FIGURE 8 ANTI-SURGE CONTROLLER RESPONSE. THE SOLID LINE REPRESENTS THE SURGE LINE, WHILE THE DOTTED LINE REPRESENTS THE BEST OPERATING LINE.

followed over the sample time. The first compressor approaches surge at one point, but the anti-surge controller turns on and drives the system back to the best operating line. The second compressor system doesn't operate anywhere near the surge line for a majority of the sample time, and only begins to approach surge towards the end of the run. The anti-surge performance of the controller functions as expected, due mostly to the fact that the recycle valve PI controller works in tandem with the MPC controller. As mentioned previously, the performance of the MPC controller overall could be increased by implementing the anti-surge control in the MPC rather than as a separate controller.

Sensitivity Analysis

Table 1 below shows the results of the sensitivity analysis. It appears that neither of the compressor flows vary much with torque manipulations. The discharge pressure of compressor 1 is affected by torque 1, and inversely affected by torque 2. The discharge pressure of compressor 2 is affected very little by torque 1, due mostly to the intermediate tank that separates the 2 compressor systems and decouples them.

TABLE 1 SENSITIVITY ANALYSIS PERFORMED IN APMONITOR

APMonitor		Objective	CV(1)	CV(2)	CV(3)	CV(4)	CV(5)
	Sensitivities	Function	p(5).n(4).flow1	p(5).n(4).flow2	p(5).n(4).p_out1	p(5).n(4).p_out2	p(5).n(4).presstank
MV(1)	p(1).n(1).torq[1]	0.00	-1.311E-05	6.793E-05	8.83492	9.246E-03	0.787828
MV(2)	p(1).n(1).torq[2]	0.00	2.573E-05	2.446E-04	-14.871	1.16248	-1.65396

Conclusions and Recommendations

As shown in the disturbance rejection and set point tracking figures, the system can drift considerably from the set point. The root cause of the drifting value is due to the inherent nonlinearity of the system. The controller implemented is a linear model predictive controller, and any significant deviation from the steady state values will result in inaccurate predictions. Recommended future work includes developing a nonlinear MPC that integrates anti-surge and pressure set point control. This would not only increase the set point tracking and disturbance rejection performance of the controller, but would allow for a faster solve time on the computer. This is a recommended step before this controller is implemented in a physical pilot system and validated with experimental data.

Acknowledgments

The authors would like to thank Dr. Mehmet Mercangöz, as well as Dr. John Hedengren for their continual help and mentorship throughout the project.

References

1. Cortinovis, A., Ferreau, H., Lewandowski, D., & Mercangöz, M. (2015). Experimental evaluation of MPC-based anti-surge and process control for electric driven centrifugal gas compressors. Journal of Process Control, 34, 13-25. doi:10.1016/j.jprocont.2015.07.001

Appendix

State Space Model 1 – Compressor 1 flow

$$\text{Torque 1 states}\ \ [\dot{x}_1] = A[x_1] + B[\tau_1]$$

$$\text{Torque 2 states}\ \ [\dot{x}_2] = C[x_2] + D[\tau_2]$$

$$\text{Compressor 1 flow}\ \ [q_1] = E[x_1] + F[x_2] + q_{1,0}$$

$$A = \begin{bmatrix} -0.1216 & -0.2787 & -0.1963 & 0.1248 \\ -0.1554 & -0.5764 & -0.6972 & 1.3509 \\ 0.1244 & 0.0656 & -1.1303 & 10.9798 \\ 0.5790 & 1.1591 & -5.3835 & -2.8584 \end{bmatrix}$$

$$B = [0.0652 \quad 0.3301 \quad 2.3957 \quad -2.0993]$$

$$C = \begin{bmatrix} -0.0322 & -0.0608 \\ -0.0106 & -0.1002 \end{bmatrix}$$

$$D = [0 \quad -0.0015]$$

$$E = [0.1995 \quad -0.0024 \quad -0.0016 \quad -0.0005]$$

$$F = [0.2426 \quad -0.0008]$$

$$q_{1,0} = 0.594$$

State Space Model 2 – Compressor 2 flow

$$\text{Torque 1 states}\ \ [\dot{x}_1] = A[x_1] + B[\tau_1]$$

$$\text{Torque 2 states}\ \ [\dot{x}_2] = C[x_2] + D[\tau_2]$$

$$\text{Compressor 2 flow}\ \ [q_2] = E[x_1] + F[x_2] + q_{2,0}$$

$$A = \begin{bmatrix} -0.1181 & -0.2726 & -0.9155 & 0.6859 \\ -0.1317 & -0.2478 & -3.5097 & 0.8864 \\ -0.4582 & 4.3268 & -0.4612 & 0.1865 \\ 0.2484 & -6.0631 & 1.4289 & -7.3154 \end{bmatrix}$$

$$B = [0.0161 \quad -0.0073 \quad -0.0032 \quad 0.2808]$$

$$C = \begin{bmatrix} -0.1243 & 0.3834 & -0.7363 & -0.8367 \\ 0.1976 & 0.1730 & -3.4309 & -0.4452 \\ 0.3762 & 3.8950 & -0.6471 & -0.1561 \\ 1.6871 & 21.0198 & -3.3491 & -20.4480 \end{bmatrix}$$

$$D = [0.0813 \quad -0.1645 \quad 0.0163 \quad 2.5321]$$

$$E = [-0.2097 \quad -0.0005 \quad -0.0118 \quad -0.0061]$$

$$F = [0.2404 \quad -0.0016 \quad -0.0089 \quad -0.0044]$$

$$q_{2,0} = 0.594$$

State Space Model 3 – Compressor 1 discharge pressure

$$\text{Torque 1 states } [\dot{x}_1] = A[x_1] + B[\tau_1]$$

$$\text{Torque 2 states } [\dot{x}_2] = C[x_2] + D[\tau_2]$$

$$\text{Compressor 1 discharge pressure } [p_1] = E[x_1] + F[x_2] + p_{1,0}$$

$$A = \begin{bmatrix} -0.0005 & 0.0022 \\ -0.0009 & -0.0175 \end{bmatrix}$$

$$B = [0.000143 \quad 0.000928]$$

$$C = \begin{bmatrix} -0.0027 & -0.012 \\ 0.0261 & -0.1419 \end{bmatrix}$$

$$D = [0 \quad 0.0031]$$

$$E = [278920 \quad -300], F = [342130 \quad -230]$$

$$p_{1,0} = 168885$$

State Space Model 4 – Compressor 2 discharge pressure

$$\text{Torque 1 states } [\dot{x}_1] = A[x_1] + B[\tau_1]$$

$$\text{Torque 2 states } [\dot{x}_2] = C[x_2] + D[\tau_2]$$

$$\text{Compressor 2 flow } [p_2] = E[x_1] + F[x_2] + p_{2,0}$$

$$A = [-2.431 \times 10^{-6}]\,, B = [-1.41 \times 10^{-8}], C = [-1.99 \times 10^{-5}], D = [1.65 \times 10^{-6}]$$

$$E = [-2624900], F = [2823400], p_{2,0} = 190000$$

State Space Model 5 – Intermediate tank pressure

$$\text{Torque 1 states } [\dot{x}_1] = A[x_1] + B[\tau_1]$$

$$\text{Torque 2 states } [\dot{x}_2] = C[x_2] + D[\tau_2]$$

$$\text{Intermediate Tank Pressure } [p_{inter}] = E[x_1] + F[x_2] + p_{inter,0}$$

$$A = \begin{bmatrix} -0.0008 & -0.0014 \\ 0.0029 & -0.0084 \end{bmatrix}$$

$$B = [9.7 \times 10^{-6} \quad 3.04 \times 10^{-4}]$$

$$C = \begin{bmatrix} -0.0213 & 0 \\ 1 & 0 \end{bmatrix}$$

$$D = [1 \quad 0]$$

$$E = [241320 \quad -170], F = [-6.352 \quad -0.1716]$$

$$p_{inter,0} = 155000$$