Optimization of a Simply Loaded Reinforced Concrete Beam

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ME 575 – Optimization Techniques

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Executive Summary

This project is for the design of a reinforced concrete beam subject to a uniform distributed load. The objective was to minimize the cost of the beam subject to constraints on the percent of steel, the strain, the capacity of the beam and the geometry of the beam.

The Problem

The goal of this optimization project is to minimize the cost of a simply supported reinforced concrete beam subject to a uniform distributed load. To understand this problem it is important to know a little about reinforced concrete theory.

When a beam is subject to a uniform load, it bends creating compression in the top and tension in the bottom. While concrete is quite strong in compression, it is weak in tension. Thus steel bars are added to the beam to support the tension. The Whitney rectangular stress distribution (Figure 1) shows the idealized model of a beam in this condition.

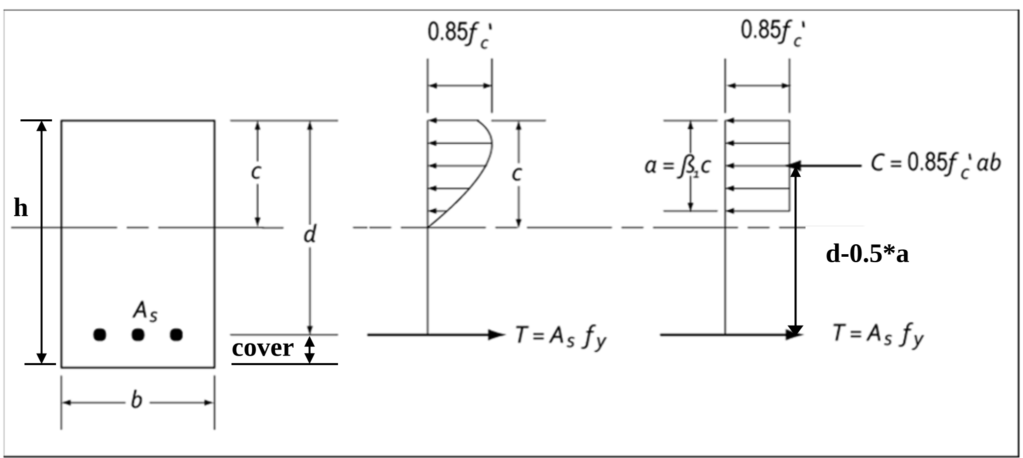

Figure 1: Whitney rectangular idealization of a reinforced concrete beam.

Capacity

The moment capacity of the beam is found by taking the moment created by the tensile and compressive forces *C* and *T*. The moment arm is the distance between where the forces act. A reduction factor φ is used as a safety factor. In this case φ = 0.9.

$$\varphi M_n = \varphi \left(T * d - \frac{a}{2}\right)$$

$$T = A_s f_y$$

Where A_s is the area of steel and f_y is the yield strength of the steel and *b, h, d* are the dimensions of the beam as shown in Figure 1.

Demand

The demand is based on the loads on the beam including the weight of the beam as well as dead and live loads. The demand we are concerned with is the maximum bending moment which is

$$M_u = \frac{wl^2}{8}$$

Where *w* is the total distributed load and *l* is the length as shown in the Figure 2.

The factored load w is found in the equation below where w_b is the weight of the beam per inch, w_c is the density of concrete in lbs per cubic inch, *ll* is the given live load per inch, and *dl* is the given dead load per inch The factored load represents the worst case scenario.

$$w = 1.6(ll) + 1.2(dl + wb)$$

$$w_b = w_c * (b * h)$$

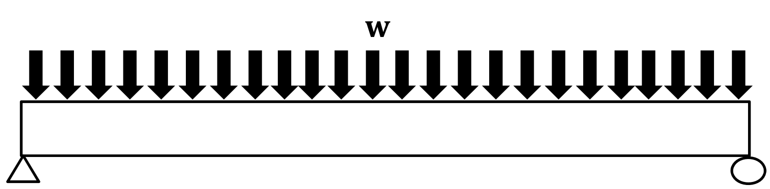

Figure 2: Simply supported beam subject to a uniform distributed load.

Strain

Another goal of beam design is to have the beam in tension control meaning that the steel yields before the concrete cracks. This occurs when the strain (ε_t) is greater than 0.005.

$$\varepsilon_t = \frac{d-c}{d} * 0.003 \geq 0.005$$

$$d = h - cover$$

$$c = \frac{a}{\beta_1}$$

$$a = \frac{A_s * f_y}{0.85 * f_c' * b}$$

$$\beta_1 = 0.850 - 0.008\left(\frac{f_c' - 4000}{1000}\right) * 0.005 \geq 0.65$$

Where d is the distance from the top of the beam to the center of the reinforcement, c is the actual height of the beam in compression, a is the idealized height in compression and β_1 is a property factor based on the compressive strength of concrete (f_c'). The value of f_c' selected for this problem of 4000 is greater than 0.65.

Steel Percentage

Another consideration in beam design is the percent of steel in the beam ρ. It has both a lower bound found in the equation below and an upper bound found empirically in tables. The max steel percentage is closely related to the allowable strain.

$$\rho = \frac{A_s}{b * d}$$

$$\rho_{min} = \frac{3\sqrt{f_c'}}{f_y}$$

Other Constraint Considerations

In order to avoid local buckling and shear failure, beams need to be rather stout. Common values for the height are 1 to 3 times the width. For this problem we choose to constrain the height to be less than or equal to twice the width.

$$h \leq 2b$$

The last constraint we placed on the problem was the weight of the beam. For the sake of movement of the beam to the construction site we decided to constrain the weight of the beam to 50 lbs/in.

$$w_b < 50 \text{ lbs/in}$$

Model Parameters

The objective of this model is to minimize the price of the concrete subject to constraints on the percent of steel, the strain, the capacity of the beam and the geometry of the beam. The parameters in the following table were chosen based on common values in reinforced concrete design. All the units for this problem are in pounds and inches.

Table 1: Parameter Values

Parameter	**Value**
Yield Strength of Steel, f_y	60000 psi
Compressive Strength of Concrete, f_c'	4000 psi
Reduction factor, φ	0.9
Density of Concrete, w_c	0.08681 lb/in^3
Dead Load, *dl*	100 lbs/in
Live Load, *ll*	200 lbs/in
Length, *l*	240 in
Price of Concrete	0.003215 \$/in^3
Price of Steel	0.1132 \$/in^3
Max. Percent of steel, ρ_{max}	0.0181
Cover Distance (h-d)	3 in