ME 575: Project 1 – Compliant Knee Simulator

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Background

Your professor is developing a knee simulator to replicate the motion and feel of a MCL (a ligament in the knee) sprains. A compliant cantilever beam (rectangular) will be used to simulate the motion of the knee. Your professor has asked you to optimize the parameters and material of the beam that will result in an appropriate force at the end displacement. *The objective is to **minimize the length** of the cantilever beam so that the mechanism will fit within the knee simulator.*

Parameters

The variables defining the cantilever beam are L, w, t, E, σ_{ut}, and σ_y.

L = Length of Beam
w = Width of Beam
t = Thickness of Beam
E = Modulus of Elasticity of Material

σ_{ut} = Ultimate Strength of Material
σ_y = Yield Strength of Material
S_f' = Uncorrected Fatigue Strength

The materials available for construction are steel and aluminum which have the following properties:

4130 Q&T@400 Steel	7075T6 Aluminum
E = 207 GPa	E = 71.7 GPa
σ_{ut} = 1627 MPa	σ_{ut} = 572 MPa
σ_y = 1462 MPa	σ_y = 503 MPa
S_f' = 830 MPa	S_f' = 290 MPa

Psuedo-Rigid-Body Model

In this problem we will model the cantilever beam following the pseudo-rigid body model (PRBM) described in Compliant Mechanisms[a]. The PRBM is a method of solving large deflection problems by modeling the beam as a spring and pin joint with a rigid link.

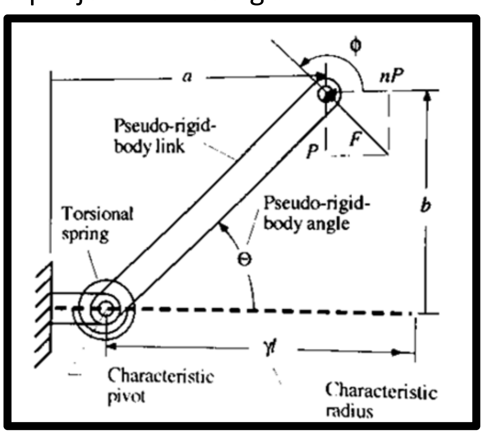

Figure 1: Psuedo-Rigid-Body Model of a Cantilever Beam[a]

a. L. Howell, *Compliant Mechanisms,* John Wiley & Sons, Inc., New York, 2001

The horizontal (a) and vertical (b) positions of the beam after displacement are given as follows:

$$a = L\big(1 - \gamma(1 - \cos\Theta)\big)$$

$$b = \gamma L \sin\Theta$$

Θ is the PRBM angle and γ is the characteristic radius factor found using the following:

$$\gamma = 0.852144 - 0.0182867n$$

n is the ratio of the vertical to horizontal force and needs to be between 0 and .5 for the equation above to be valid. For the following equations to be valid, the applied force (F) is assumed to always be perpendicular to the beam throughout the motion.

$$n = -\frac{1}{\tan\phi}$$

where

$$\phi = \frac{\pi}{2} + \Theta$$

P is the vertical component of the applied force and is found using:

$$P = \frac{K\Theta}{\gamma l(\cos\Theta + n\sin\Theta)}$$

I is the moment of inertia, and in the case of a rectangular beam is:

$$I = \frac{wt^3}{12}$$

K is the PRBM spring constant that appears due to the bending of the beam.

$$K = \gamma K_{\Theta}\frac{EI}{L}$$

where

$$K_{\Theta} = 2.654855 - 0.0509896n + 0.0126749n^2 - 0.00142039n^3 + 0.0000584525n^4$$

Failure Prevention

The beam must be designed such that it doesn't yield in static or fatigue loading. A safety factor (SF) of 1.2 needs to be used when solving for both static and fatigue failure.

The absolute maximum stress (σ_{max}) the beam will experience in a static load is:

$$\sigma_{max} = \frac{P(a + nb)c}{I} + \frac{nP}{wt}$$

with

$$c = \frac{t}{2}$$

and the minimum stress (σ_{min}) = 0. The maximum stress times the safety factor (SF) must be less than the yield stress of the material. The modified-Goodman criteria for fatigue is given as:

$$\frac{1}{SF_{fatigue}} = \frac{\sigma_{alt}}{s_f} + \frac{\sigma_{mean}}{\sigma_{ut}}$$

with

$$\sigma_{alt} = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}$$

s_f is the fatigue strength after applying various correction factors.

$$s_f = c_{surf} c_{size} c_{load} c_{reliab} c_{misc} {s_f}'$$

In this loading situation:

c_{surf}	0.95
c_{size}	1.0
c_{load}	1.0
c_{reliab}	.70
c_{misc}	1.0

$SF_{fatigue}$ must be greater than or equal to the specified safety factor (SF) of the overall design.

Additional Constraints

For the knee simulator to be useful it needs to have a force (F) of 90 N at a PRBM displacement angle (Θ) of 7°. It is important to remember that Θ must be in radians. Note that the force (F) needed to move the beam to the desired angle is given by: $F = P\sqrt{n^2 + 1}$

To fit within the knee simulator, the beam dimensions must be less than the following: .5m long, 10cm wide, and 1cm thick. Due to manufacturing constraints, the thickness must be greater than 0.8 mm and the width must be greater than 1 cm.

Report

Turn in a report (3 pages or less) that gives the main optimization results. Include the following:

- A table summarizing the optimal solution
- A brief introduction to the problem and the model
- A brief description of the results of optimization
- A brief discussion on the meaning of the results (Do you feel you found a global optimum?)
    - Be sure to discuss any differences between steel and aluminum
- Appropriate plots showing the optimum solution and the design space around the optimum
- Provide a listing of your model in the appendix.