Branch and Bound Exercise

Consider the optimization problem with the following objective function and constraints.

$$\begin{aligned} \min\ & 4x_1^4 - 4x_2x_1^2 + x_2^2 + x_1^2 - x_1 + 1 \\ s.t.\ & -1 \le x_1 \le 1 \\ & -1 \le x_2 \le 2 \end{aligned}$$

1. Verify that [0.5,0.5] is optimal for the relaxed solution (with continuous variables). This is the root node, lower bound for the integer solution objective, and starting point for branch and bound.

2. For x_1 constrained to integer values (-1, 0, 1) and x_2 constrained to integer values (-1, 0, 1, 2), determine the optimal solution by branch and bound. Compare the number of optimization evaluations from branch and bound to an exhaustive search.