Worksheet on Interior Point Methods

$$\begin{aligned} \min_{x \in R^2} \quad & x_2(5 + x_1) \\ s.t. \quad & x_1 x_2 \geq 5 \\ & x_1^2 + x_2^2 \leq 20 \end{aligned}$$

1. Transform the problem into standard form:

$$\begin{aligned} \min_{x \in R^n} \quad & f(x) \\ s.t. \quad & c(x) = 0 \\ & x \geq 0 \end{aligned}$$

2. Set up the barrier function to solve for a search direction in x, λ, and z.

$$\begin{bmatrix} W_k & \nabla c(x_k) & -I \\ \nabla c(x_k)^T & 0 & 0 \\ Z_k & 0 & X_k \end{bmatrix} \begin{pmatrix} d_k^x \\ d_k^\lambda \\ d_k^z \end{pmatrix} = - \begin{pmatrix} \nabla f(x_k) + \nabla c(x_k)\lambda_k - z_k \\ c(x_k) \\ X_k Z_k e - \mu_j e \end{pmatrix}$$

where $W_k = \nabla_{xx}^2 L(x_k, \lambda_k, z_k) = \nabla_{xx}^2 (f(x_k) + c(x_k)^T \lambda_k - z_k)$