ME 575: Spring Design

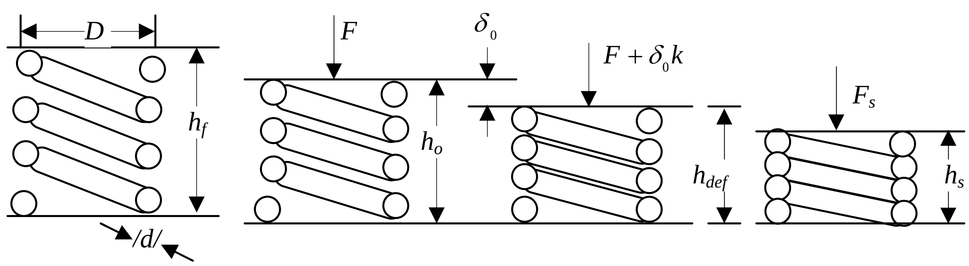
The specifications and modeling equations for compression spring design are given below. We wish to determine the spring design that maximizes the force of a spring at its preload height, h_o, of 1.0 inches. The spring is to operate an indefinite number of times through a deflection δ_o, of 0.4 inches, which is an additional deflection from h_o. The stress at the solid height, h_s, must be less than S_y to protect the spring from inadvertent damage.

The variables defining the design of a spring are d, D, n, h_o and h_f,

where
d = wire diameter
D = coil diameter
n = number of coils in the spring
h_o = preload height
h_f = free height (spring exerting no force)

and other variables/functions, as shown below, are,

δ_o = deflection from preload height
h_{def} = deflected height
h_s = solid height



The force in a linear spring is given by

$$F = k\Delta x$$

where k is the spring stiffness and Δx is the deflection.

The spring stiffness (rate) is

$$k = \frac{Gd^4}{8D^3 n}$$

where G is the shear modulus of the material. The stress in a spring with an axial load of F is,

$$\tau = \frac{8FD}{\pi d^3} K$$

where K is the Wahl factor that accounts for stress concentration due to curvature of the spring as well as direct shear:

$$K = \frac{4D - d}{4(D - d)} + 0.62\frac{d}{D}$$

Solid height, h_s, is the height at which the coils of the compressed spring close up. It is simply,

$$h_s = nd$$

If the spring is to operate indefinitely through a deflection δ_0, it must be designed so that the material does not fail in fatigue. A fatigue criterion for compression spring design is

$$\tau_a \le S_e / S_f$$

$$\tau_a + \tau_m \le S_y / S_f$$

where τ_m is the mean shear stress and τ_a is the alternating shear stress, defined to be,

$$\tau_m = \frac{\tau_{\max} + \tau_{\min}}{2} \qquad \tau_a = \frac{\tau_{\max} - \tau_{\min}}{2}$$

and where S_f is a factor of safety, S_e is the endurance limit, and S_y is the yield strength in shear. S_f and S_e are constants, but S_y is a function of material properties Q and w, according to the relation,

$$S_y = 0.44\frac{Q}{d^w}$$

Also, to be reasonable, the ratio D/d should be $4 \le D/d \le 16$. The diameters of wire considered should be $0.01 \le d \le 0.2$ inches. The maximum allowable width for the spring, i.e., $(D + d)$, is 0.75 inches. To insure that the spring does not reach solid height in service, a clash allowance of 0.05 inches should be provided. This means the solid height should be at least 0.05 inches below the lowest point of deflection the spring reaches in service.

For this problem, assume

$G = 12 \times 10^6$ psi	$S_f = 1.5$
$S_e = 45{,}000$ psi	$Q = 150{,}000$ psi
$w = 0.18$	

Also assume the number of coils is continuous for optimization. (After optimizing you can round off to the nearest integer if you wish.)

Sample Design:
(Note, no guarantee this is a feasible design)

```
wireDia              0.05000000  (in)
coilDia               0.5000000  (in)
coilNum                10.00000
freeHeight             1.500000  (in)

Spring constant                7.500 (lb/in)
Wahl Factor                    1.14533
Force at Preload height        3.75 (lb)
Alternating Stress             17499 (psi)
Mean Stress                    61248 (psi)
Yield Strength                 113170 (psi)
Clash Allowance                0.1 (in)
Diameter Sum                   0.55 (in)
```

Report (<3 pages):
Turn in a report with the following sections:

1) Title Page with Summary. The Summary should be short (less than 50 words), and give the main optimization results.
2) Procedure: Give a brief description of your model. You are welcome to refer to the assignment which should be in the Appendix. Also include:
    a. A table with the analysis variables, design variables, analysis functions and design functions.
3) Results: Briefly describe the results of optimization (values). Also include:
    a. A table showing the optimum values of variables and functions, indicating binding constraints and/or variables at bounds (highlighted)
    b. A table giving the various starting points which were tried along with the optimal objective values reached from that point.
4) Discussion of Results: Briefly discuss the optimum and design space around the optimum. Do you feel this is a global optimum? Also include and briefly discuss:
    a. A "zoomed out" contour plot showing the design space (both feasible and infeasible) for coil diameter vs. wire diameter, with the feasible region shaded and optimum marked.
    b. A "zoomed in" contour plot of the design space (mostly feasible space) for coil diameter vs. wire diameter, with the feasible region shaded and optimum marked.
5) Appendix:
    a. Listing of your model with all variables and equations

Any output from the software is to be integrated into the report (either physically or electronically pasted) as given in the sections above. Tables and figures should all have explanatory captions. Do **not** just staple pages of output to your assignment: all raw output is to have notations made on it. For graphs, you are to shade the feasible region and mark the optimum point. For tables of design values, you are to indicate, with arrows and comments, any variables at bounds, any binding constraints, the objective, etc. (You need to show me that you understand the meaning of the output you have included.)

(This problem is a modified version taken from Fox, *Optimization Methods in Engineering Design*, Addison Wesley)