Step-cone Pulley [1]

A step-cone pulley as shown below in the diagram is used to transmit a power of 0.65 horsepower.

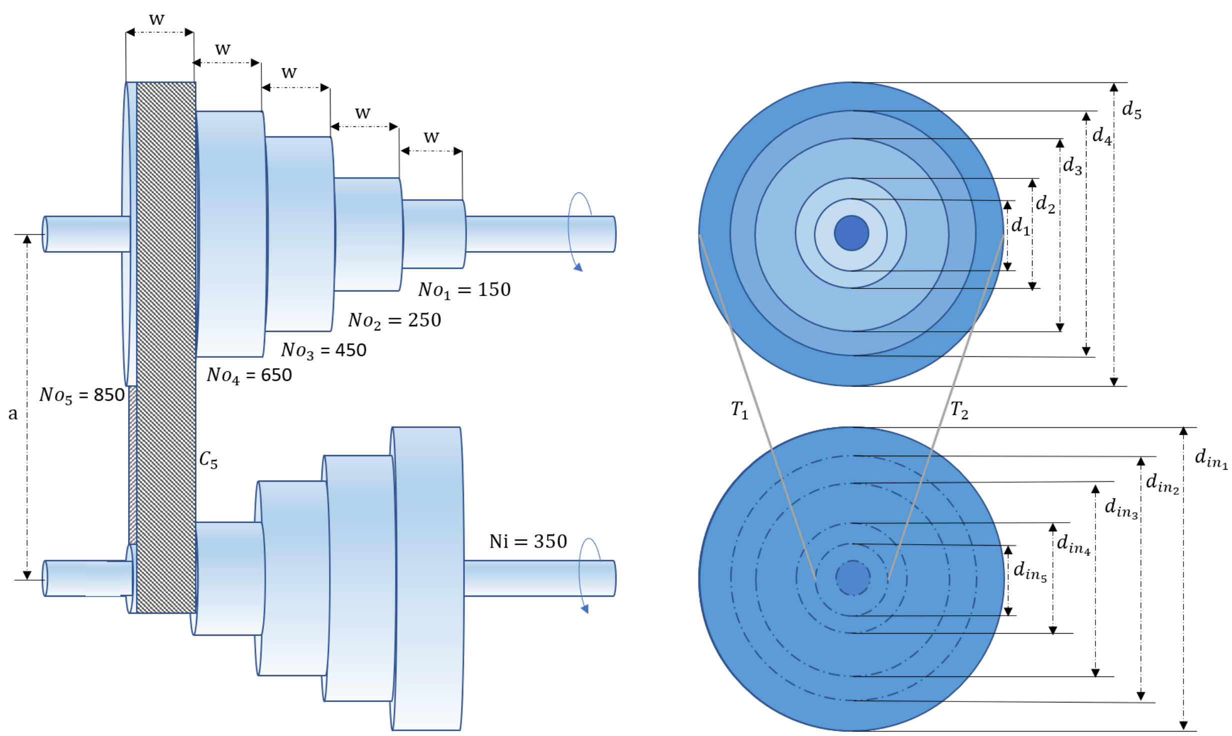

Figure 1: The pulley system from 2 viewpoints.

The pulley is made up of an aluminum alloy with the density (ρ) of 168.5 lb/ft^3. The belt used is made of Polyamide (A-2) which has a thickness (t) of 0.11 in. and a maximum stress (s) of 75000 lbf/ft^2. The coefficient of friction (μ) for this model is 0.8. The fixed distance (a) between the centers of the shafts is 1.67 ft. The tension of the tight side of the belt should be at least 2 times that of the the slack side of the belt. The input shaft has a speed of 350 rpm (Ni) and the output shaft should be capable of running at speeds of 950 rpm, 650 rpm, 450 rpm, 250 rpm, and 150 rpm. The design variables will be d_i, the diameter of the ith step, and w the width of the belt and each step. The objective of this system is to minimize the weight of the step-cone pulley.

Note

The weight is found by the following equation

$$f(X) = \rho w(\pi/4)(d_1^2 + d_2^2 + d_3^2 + d_4^2 + d_5^2 + {d_{in}}_1^2 + {d_{in}}_2^2 + {d_{in}}_3^2 + {d_{in}}_4^2 + {d_{in}}_5^2) \quad (1)$$

The diameters of the input pulley (d_{in}) are found by using the ratio between the input shaft rpm and the target output rpm.

$$d_{in\,i} = d_i \left(\frac{No_i}{N_{in}} \right) \quad (2)$$

The belt lengths required for the desired output speeds are determined by the following equation:

$$C_i \simeq \frac{\pi d_i}{2}\left(1+\frac{No_i}{N_{in}}\right)+\frac{\left(\frac{No_i}{N_{in}}-1\right)^2 d_i^2}{4a}, \qquad i=1,2,3,4,5 \tag{3}$$

The angle of the lap of the belt over the ith pulley step is:

$$\theta_i = \pi - 2sin^{-1}\left[\frac{\left(\frac{No_i}{N_{in}}-1\right)d_i}{2a}\right] \tag{4}$$

The tension on the tight side of the ith step is given by:

$$T_{1i} = stw \tag{5}$$

The tension of the loose side of the ith step is given by

$$T_{2i} = \frac{T_{1i}}{e^{\mu\theta_i}} \tag{6}$$

The belt length values are subject to the following constraints:

$$C_1 - C_2 = 0 \tag{7}$$

$$C_1 - C_3 = 0 \tag{8}$$

$$C_1 - C_4 = 0 \tag{9}$$

$$C_1 - C_5 = 0 \tag{10}$$

The constraint on the power transmitted is given by:

$$\frac{(T_{1i} - T_{2i})\pi d_{ini}(350)}{33000} \tag{11}$$

The constraint on tension ratio can be given by:

$$e^{\mu\theta_i} \geq 2 \tag{12}$$

References

[1] Singiresu S Rao. *Engineering Optimization: Theory and Practice*. John Wiley & Sons, 2009.