Control Engineering, Plant Experiments & Model Development

Brigham Young University

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Topics

- Career in Process Control
- Model Development, Experimental Design
- Introduction to My Research

Process Control

- Brings Together Knowledge
  - Process
  - Dynamics, Feedback Control
  - Instrumentation
  - Computers, Networks

- Troubleshooting Skills

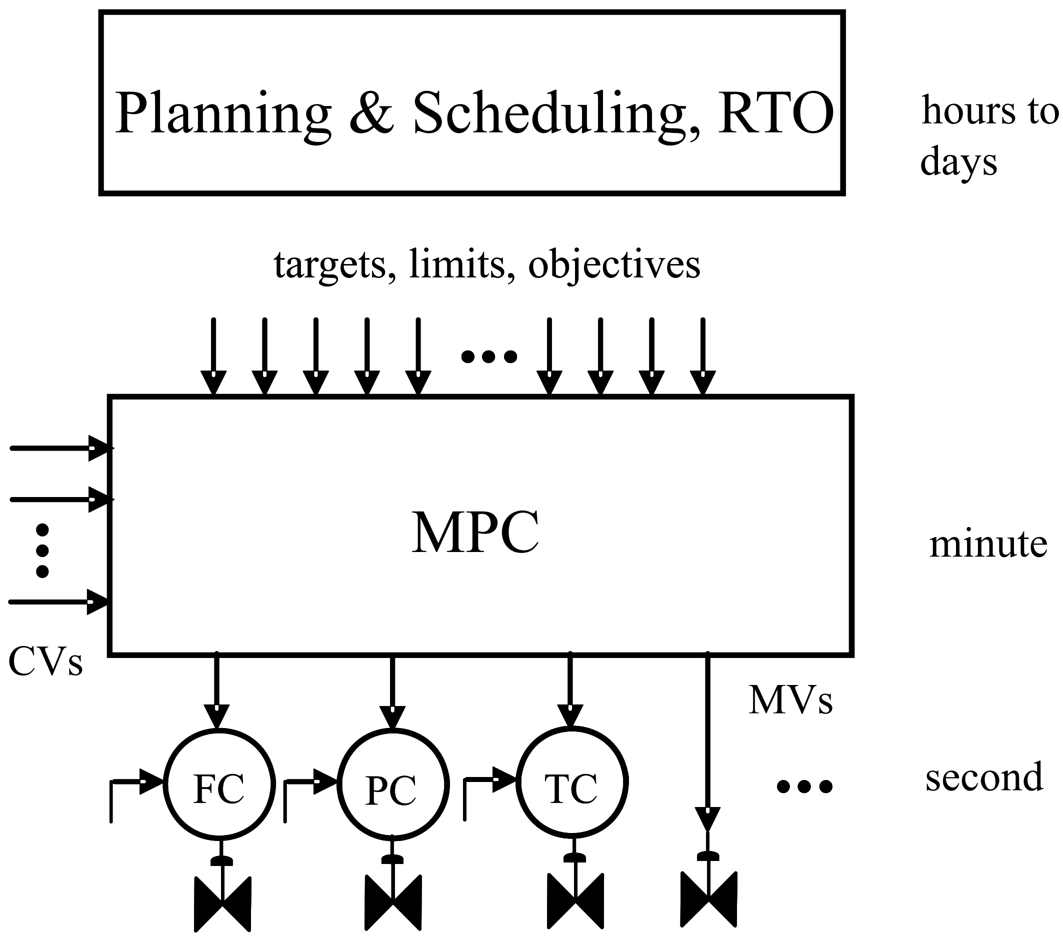
- Impacts bottom line:
  - Tighter control of product specs
  - Feed / product maximization

Importance of Statistics*

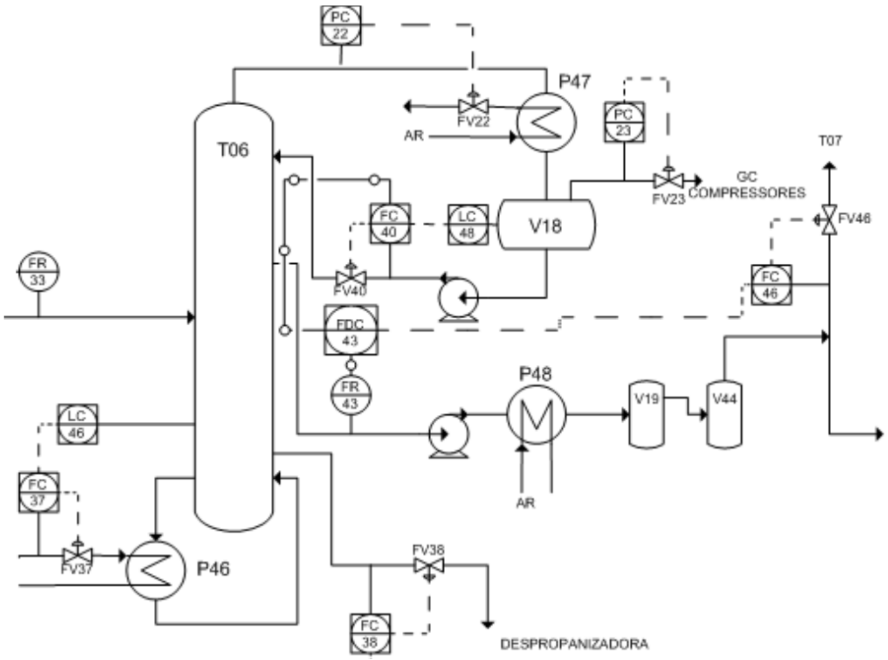
- Evaluating Means, Variance (Standard Deviation), Correlation
- Confidence Intervals
- Regression
- Hypothesis Testing
- Used in industry: 6 Sigma, Lean 6 Sigma

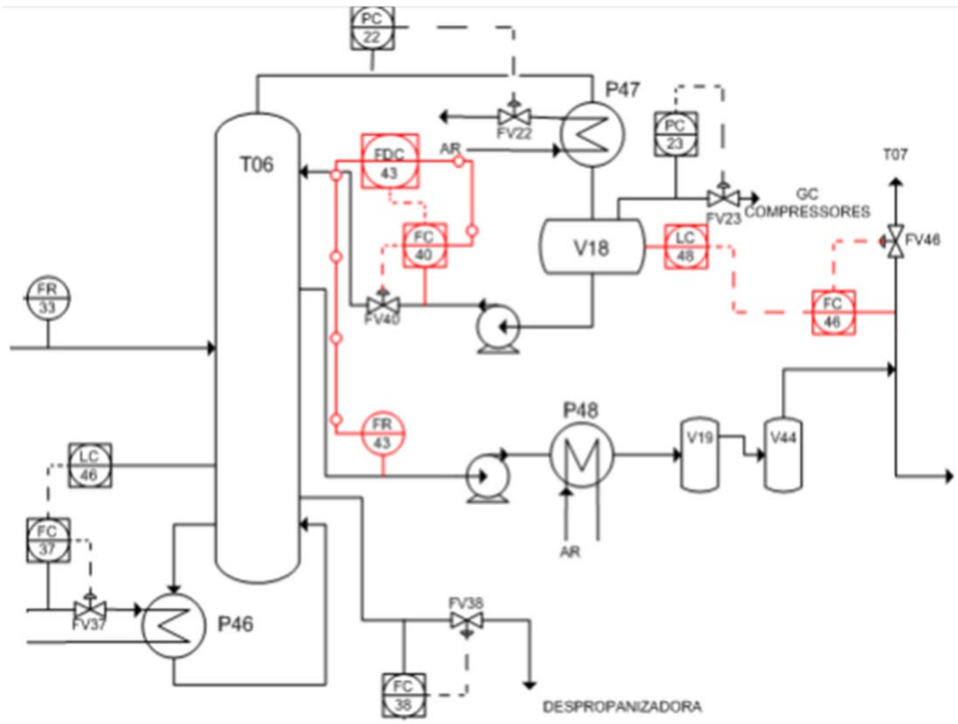
*Also recommended in Edgar, et al, “Renovating the undergraduate process control course”, *Computers and Chemical Engineering*, 30 (2006), 1749-1762.

Plant Control Hierarchy



Regulatory Control

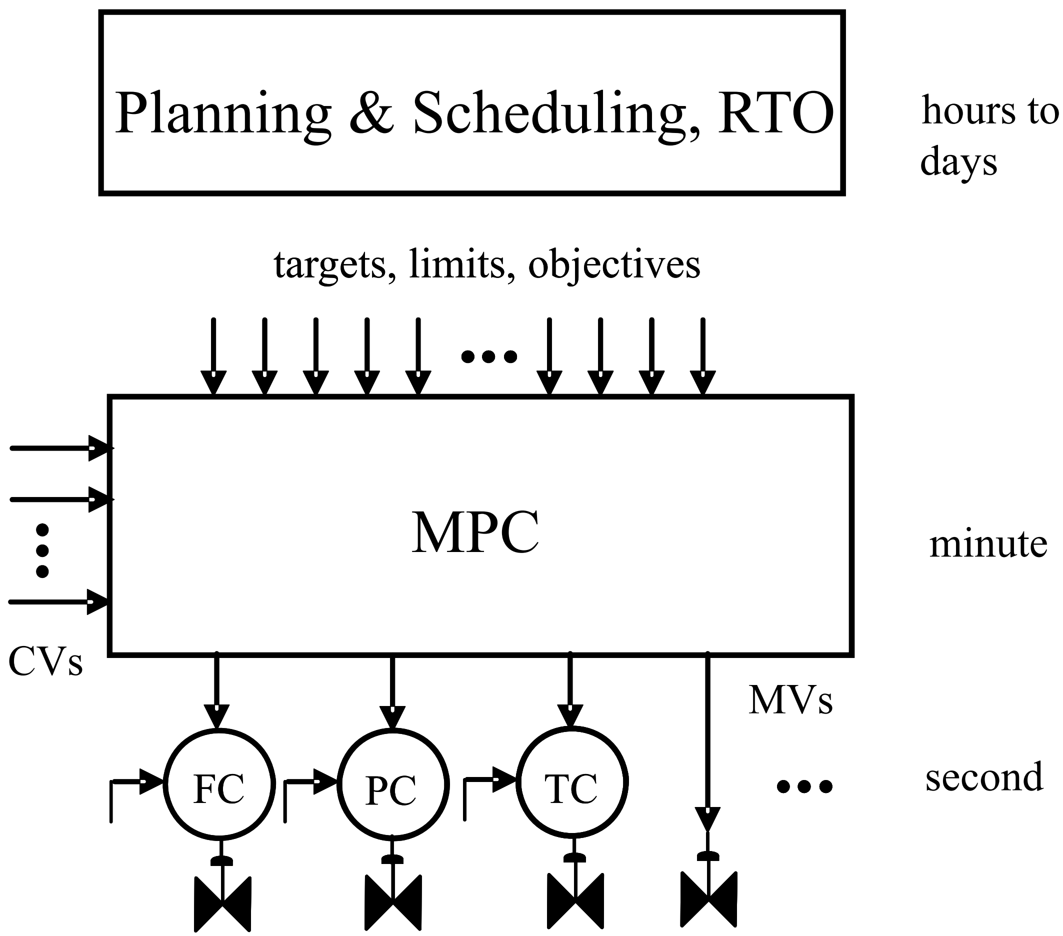




Models [Steady-State or Dynamic]

- Empirical Black Box
  - Traditional Least Squares
  - Often linear (ized)
  - → Process Control
  - Effort: Testing
  - Model must be validated
- Fundamental White Box
  - Mass, Energy, Kinetics
  - Nonlinear
  - → Design, Optimization
  - Effort: Model Building
  - Model must be validated
- Grey Box
  - Model: Fundamental & Empirical
    "Parts" Maybe Linear or Nonlinear
  - Incorporate a priori knowledge
  - No standard approach
  - → Planning, Scheduling
  - IMHO: Should use more

Plant Control Hierarchy



Fit a Steady-State Model – Least Squares

- Typical Text Book:
  - Data given (see table)
  - Fit model, e.g.,

$$y = kx + b \quad \text{find } \hat{k}, \hat{b}$$

- Industry:
  - Where / how to get data?
  - Testing required?
  - How to change process to get "best" model?
    - → Design experiment

X	Y
13.40	95.20
11.43	89.30
12.79	93.38
11.99	90.97
11.93	90.78
11.94	90.82
10.27	85.82
13.53	95.58
11.19	88.58
13.36	95.08
13.22	94.67
10.31	85.94
10.74	87.21
13.34	95.01
13.67	96.00
12.52	92.56
12.73	93.18
13.91	96.73
10.72	87.16
13.87	96.60

Consider Least Squares Estimate

$$y = ku + e$$

Least squares estimate $\sum_{i=1}^{n} (y_i - \hat{y}_i)^2$

$$\hat{k} = \frac{\sum_{i=1}^{n} y_i u_i}{\sum_{i=1}^{n} u_i^2}$$

$$\operatorname{var}(\hat{k}) = \frac{\hat{\sigma}_e^2}{\sum_{i=1}^{n} u_i^2}$$

For parameter accuracy (small parameter variance):

Design: Maximize: $|u_i|$ and/or n (experiment length)

Question: How big can we make u_i ?

Multivariable Least Squares

$$y = k_1 u_1 + k_2 u_2 + e$$

Least squares estimate:

$$\underbrace{\begin{bmatrix} u_{11} & u_{21} \\ \vdots & \vdots \\ u_{1n} & u_{2n} \end{bmatrix}}_{\mathbf{U}} \begin{bmatrix} \hat{k}_1 \\ k_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \qquad \hat{k}_1 = \frac{\left(\sum_{i=1}^{n} u_{2i}^2\right)\left(\sum_{i=1}^{n} u_{1i}^2 y_i\right) - \left(\sum_{i=1}^{n} u_{1i} u_{2i}\right)\left(\sum_{i=1}^{n} u_{2i}^2 y_i\right)}{\underbrace{\left(\sum_{i=1}^{n} u_{1i}^2\right)\left(\sum_{i=1}^{n} u_{2i}^2\right) - \left(\sum_{i=1}^{n} u_{1i} u_{2i}\right)^2}_{\det(\mathbf{U}^\mathbf{T}\mathbf{U})}}$$

For parameter accuracy (small parameter variance):

Design: Maximize: $\det(\mathbf{U}^\mathbf{T}\mathbf{U}) \rightarrow$
design size, $u_i u_j$, and/or n (experiment length)

Design of Experiments – General Problem

Min det of $\mathbf{U}^T\mathbf{U}$ (depends on model)

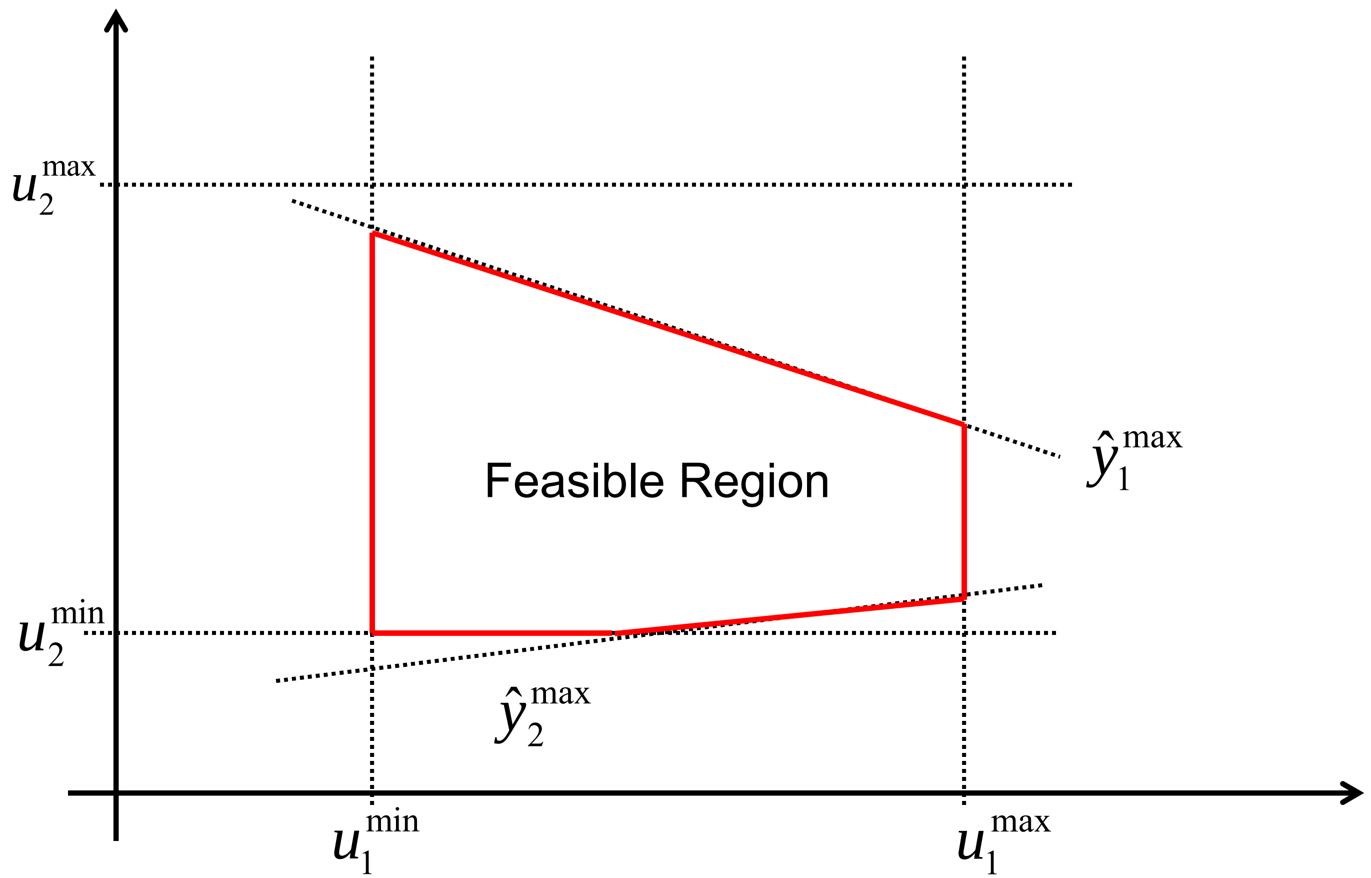
subject to:

Limits on inputs (u_i): $u_i^{\min} \le u_{ij} \le u_i^{\max}$

Limits on (predicted) outputs (y_i): $y_i^{\min} \le \hat{k}_1 u_{1i} + \hat{k}_1 u_{2i} \le y_i^{\max}$

See a problem/challenge?

Feasible Test Region

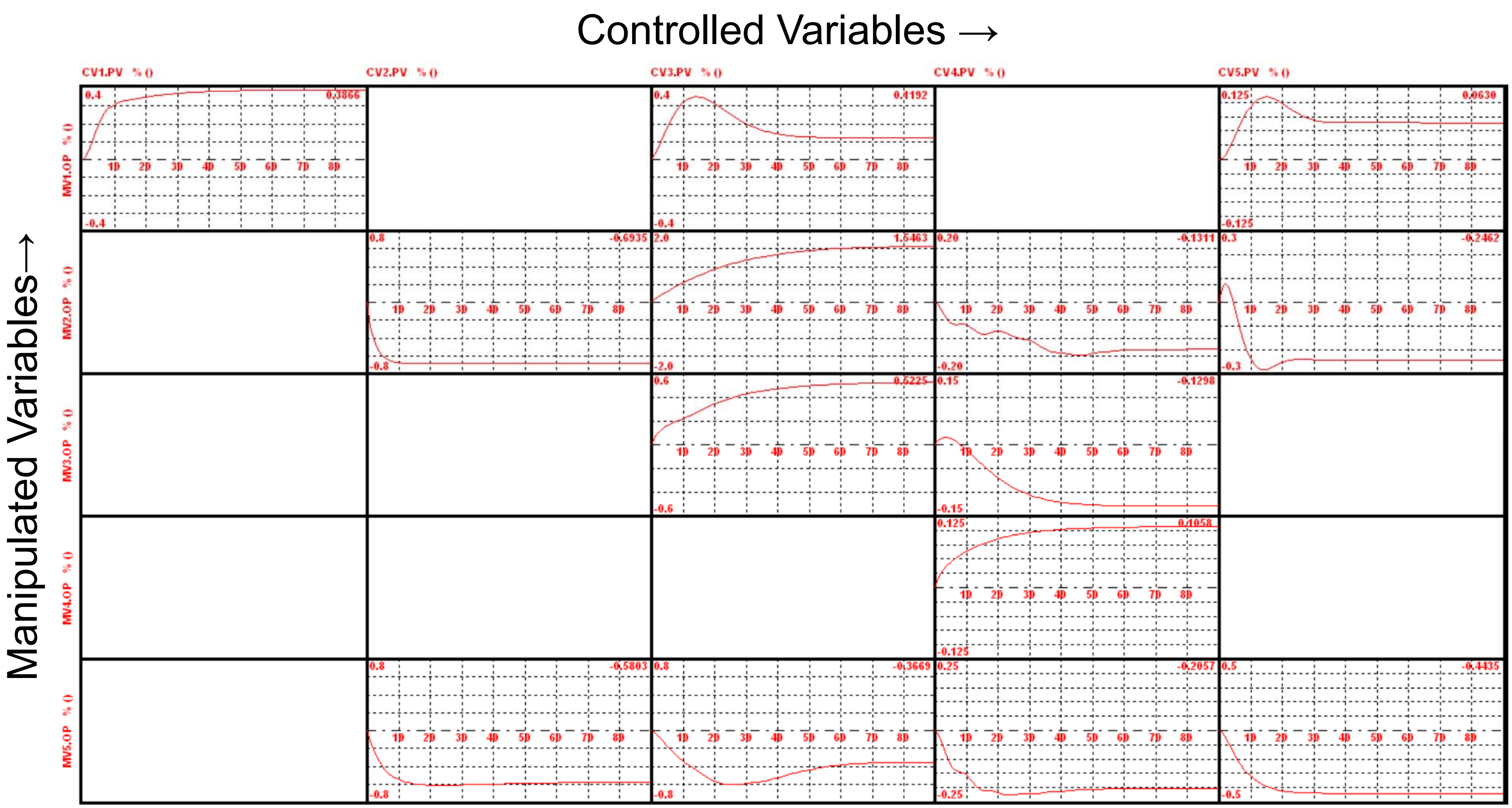


→ Maximize movement in feasible region

Design of Experiments – Practical Issues

- Mathematical formulation, → practical considerations
- Larger changes preferred
  - But (if fitting linear model) might violate linear assumption
  - To observe changes outside typical noise, disturbances
  - Will not know model well at first (to predict y), start small and adapt
- Test until parameter variance or prediction errors small

Dynamic Problem / Step Response Matrix



→ Need accurate model over time response

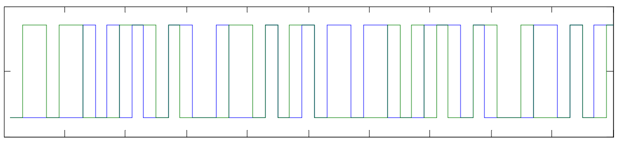
Rule of Thumb: Steps at (1/3)Tss, (2/3)Tss, (1)Tss

Experimental Design - Practice

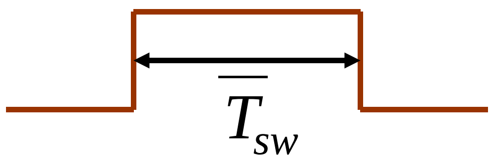
True (optimal) design not performed

In practice: Uncorrelated binary signals

- PRBS or GBN
  - D-optimal (parameter covariance) for FIR w/ input (only) constraints (Levin, M.J., 1960).

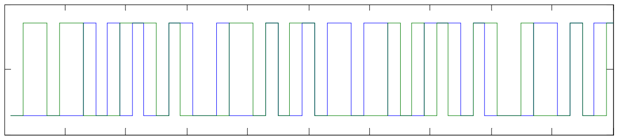


- Design variables
  - Input amplitudes*
  - Frequency* content via average
  - Experimental duration



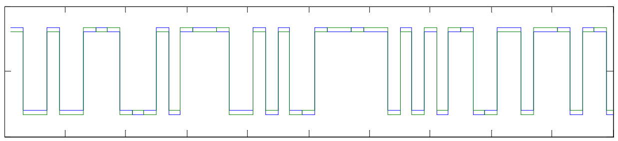
PRBS, GBN Signals – BUT!

- Suboptimal when output constraints present!



- Model parameter (or prediction) accuracy: not sufficient to
  - Guarantee control performance
  - or Even closed-loop stability

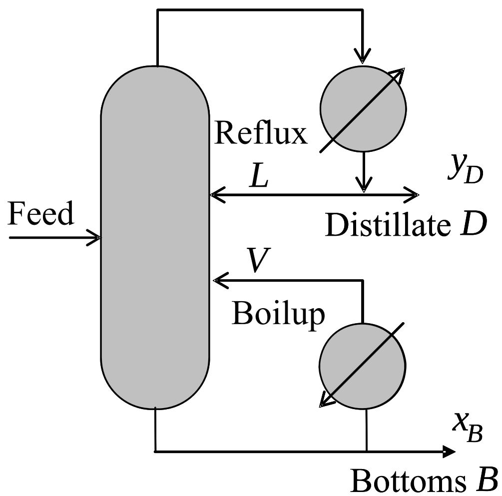
- Can be problematic for multivariable systems
  - Research since 90's: ill-conditioned systems [Anderson & Kummel (1992), Koung & MacGregor (1994), Li and Lee (1996), Featherstone & Braatz (1997), Bruwer &MacGregor (2007)]
    - Correlated inputs useful
      (generated in open or closed loop)

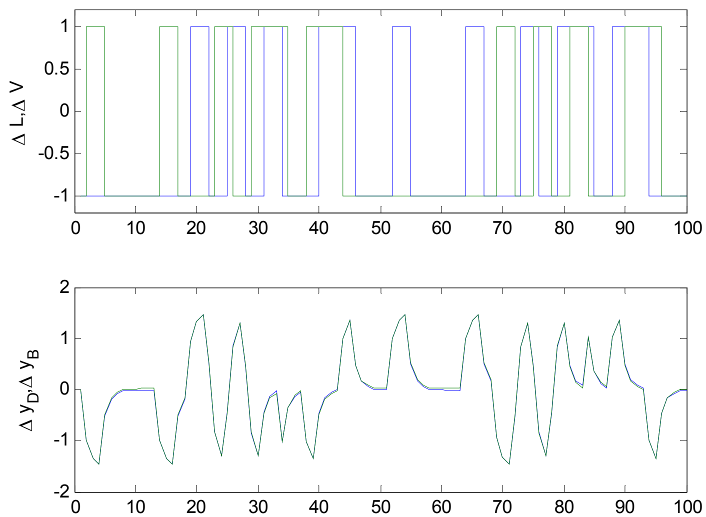


Motivating Example
Ill-Conditioned Systems: PRBS Inputs

Example: High purity distillation in LV configuration

$-1 \leq y_1, y_2 \leq 1$





Identified model (likely) leads to unstable control

Ill-Conditioned Systems: Rotated Inputs

Koung & MacGregor (1994)

Use latest $\hat{\mathbf{G}}: \mathbf{y} = \hat{\mathbf{U}}\hat{\boldsymbol{\Sigma}}\underbrace{\hat{\mathbf{V}}^T\mathbf{m}}_{\equiv \xi}$

Target: $\mathbf{V}, \sigma_{\min} (2x2)$

(favorable for IC)

ξ = rotated inputs

$$\frac{\mathrm{var}(\xi_j)}{\mathrm{var}(\xi_k)} = \frac{\hat{\sigma}_k^2}{\hat{\sigma}_j^2}$$

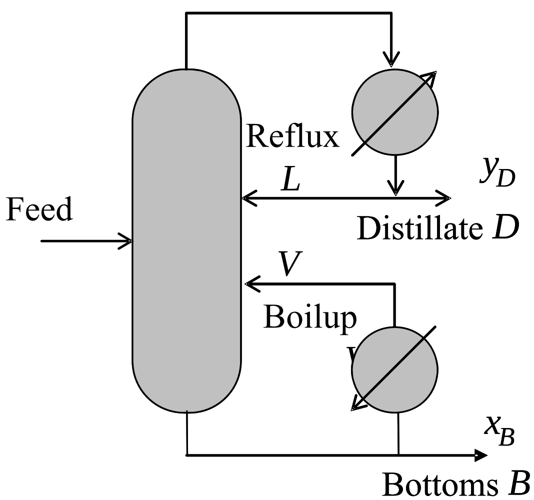
$$\mathrm{cov}(\xi_i, \xi_j) = 0, \; i \neq j$$

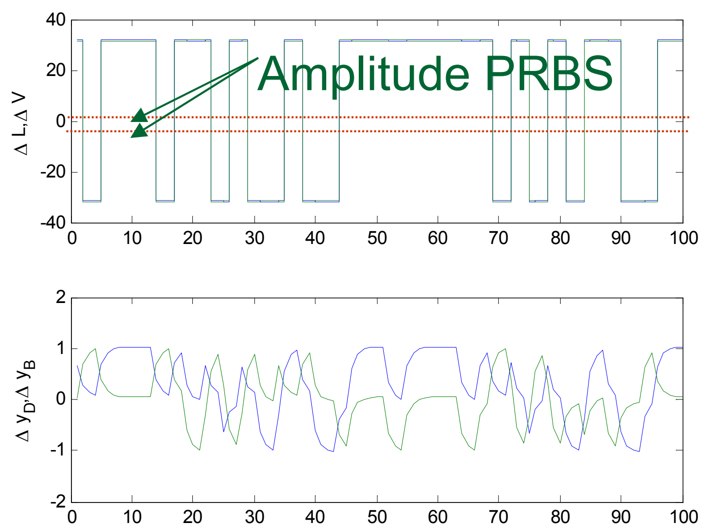
PRBS design - Amplitudes for ξ:

$$\mathbf{y}^{low} \leq \mathbf{y} = \underbrace{\hat{\mathbf{u}}_1\hat{\sigma}_1}_{\text{constant}}\xi_1 + \cdots + \hat{\mathbf{u}}_n\hat{\sigma}_n\xi_n \leq \mathbf{y}^{high}$$

- Frequency content: from estimated dynamics

Implement $\mathbf{m} = \hat{\mathbf{V}}\xi$



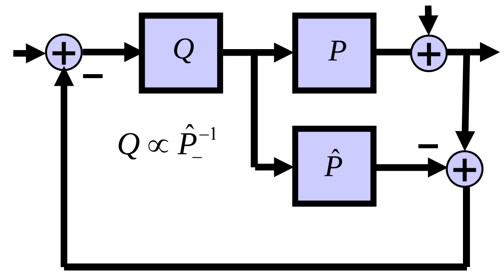


But: (highly) correlated inputs require a more accurate model

Identified model (likely) leads to stable control

Closed-loop Stability: Integral Controllability

- For Internal Model Control
  (Square *nxn* Case)



IMC Structure

Garcia & Morari (1985) - IC Condition

$$\mathrm{Re}\left[\lambda\left(\mathbf{G}\hat{\mathbf{G}}^{-1}\right)\right] > 0 \Leftrightarrow$$ There exists a robustly stabilizing controller with integral action

$\mathbf{G}$ = steady-state gain matrix, true plant $\hat{\mathbf{G}}$ = steady-state gain matrix, estimate

- Importance of Model AND Model Inverse in ID experiments
  Koung & MacGregor (1994), Li and Lee (1996)

- My Research: Online optimal design with IC condition, model uncertainty st constraints→
  solve for optimal magnitudes and correlation of inputs

Illustration: Steady-State 2x2 Model

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \hat{k}_{11} & \hat{k}_{12} \\ \hat{k}_{21} & \hat{k}_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Open Loop

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} \hat{k}_{11} & \hat{k}_{12} \\ \hat{k}_{21} & \hat{k}_{22} \end{bmatrix}^{-1} \begin{bmatrix} y_1^{sp} \\ y_2^{sp} \end{bmatrix}$$

Closed Loop

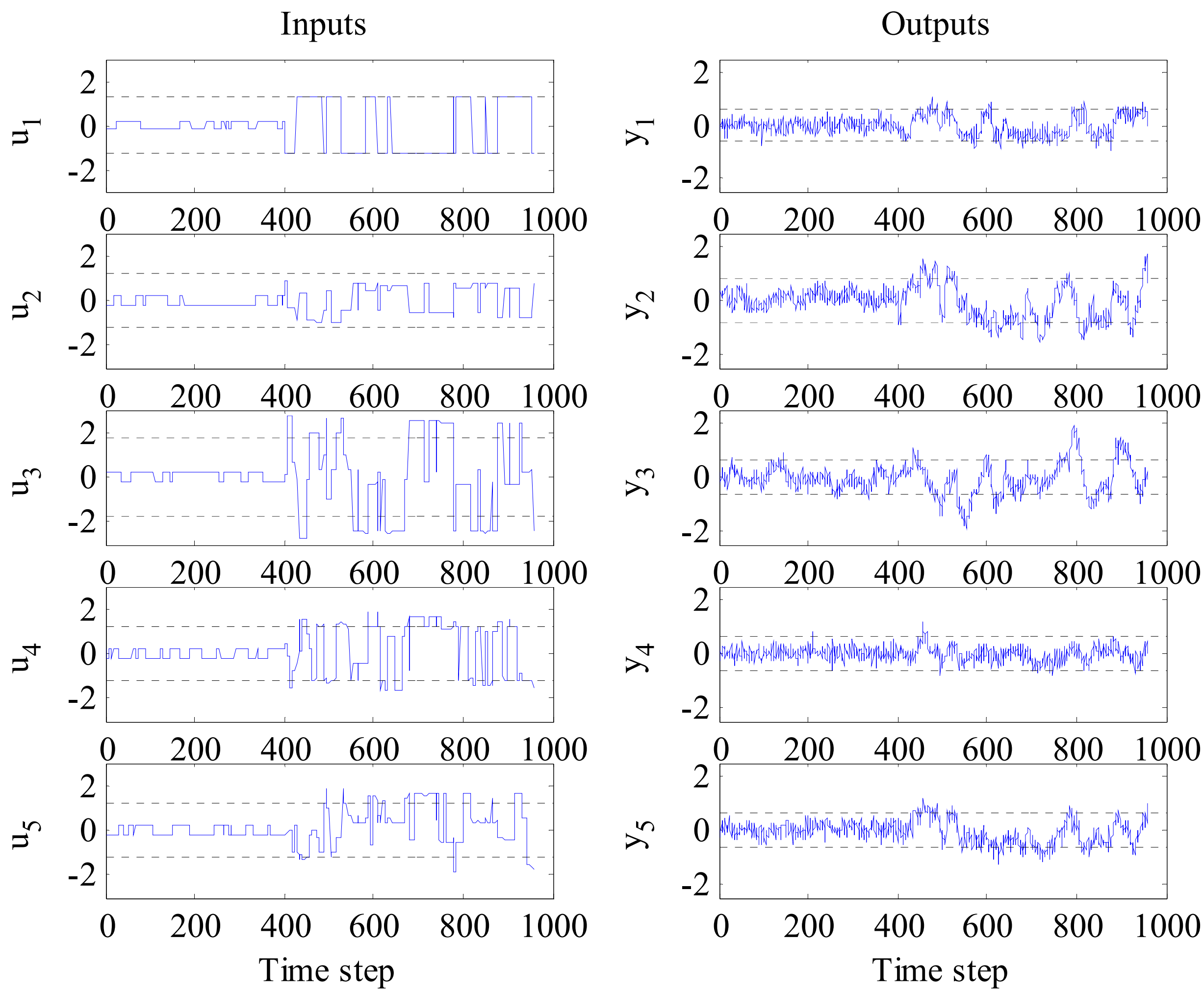
or

$$u_1 = \frac{\hat{k}_{22} y_1^{sp} - \hat{k}_{21} y_1^{sp}}{\hat{k}_{11}\hat{k}_{22} - \hat{k}_{12}\hat{k}_{21}}$$

$$u_2 = \frac{-\hat{k}_{21} y_1^{sp} + \hat{k}_{11} y_1^{sp}}{\hat{k}_{11}\hat{k}_{22} - \hat{k}_{12}\hat{k}_{21}}$$

- Each closed-loop gain depends on all open loop gains
- Accuracy of individual gains may not be sufficient

My Research: Illustration



Increase allowable amplitudes and allowable correlation as model improves

That’s It!

Thank you, Questions?