Name___________________

Practice Exam #3

Chemical Engineering 436 (Section 1)
Professor John Hedengren
Closed Book, 3 Pages of Notes, Single Sided or 1.5 Pages Front/Back

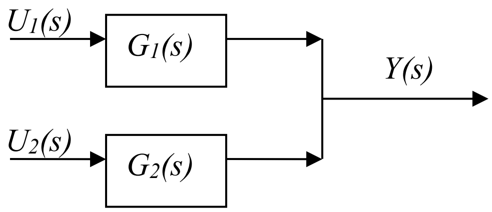
Time Limit: 50 minutes

Hints on getting more points:
1. If you do not get all the way through a problem, please indicate the exact equations and solution method that you would use.
2. More points will be given if your answers are legible and easy to read.
3. *Circle or box your final answer.*
4. Pace yourself with recommended times for each problem
    Problem 1 (30 pts) ~ 15 minutes
    Problem 2 (30 pts) ~ 15 minutes
    Problem 3 (40 pts) ~ 20 minutes

f(t) in Time Domain	*F(s)* in Laplace Domain
$\delta(t)$ unit impulse	1
$S(t)$ unit step	$\frac{1}{s}$
t ramp with slope $=1$	$\frac{1}{s^2}$
t^{n-1}	$\frac{(n-1)!}{s^n}$
e^{-bt}	$\frac{1}{s+b}$
$1-e^{-t/\tau}$	$\frac{1}{s(\tau s+1)}$
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$
$\frac{1}{\tau_1-\tau_2}\left(\exp\left(-t/\tau_1\right)-\exp\left(-t/\tau_2\right)\right)$	$\frac{1}{(\tau_1 s+1)(\tau_2 s+1)}$
$\frac{1}{\tau^n(n-1)!}t^{n-1}\exp\left(-\frac{t}{\tau}\right)$	$\frac{1}{(\tau s+1)^n}$
$\frac{1}{\tau\sqrt{1-\zeta^2}}\exp\left(-\frac{\zeta t}{\tau}\right)\sin\left(\sqrt{1-\zeta^2}\frac{t}{\tau}\right)$	$\frac{1}{\tau^2 s^2+2\zeta\tau s+1}$

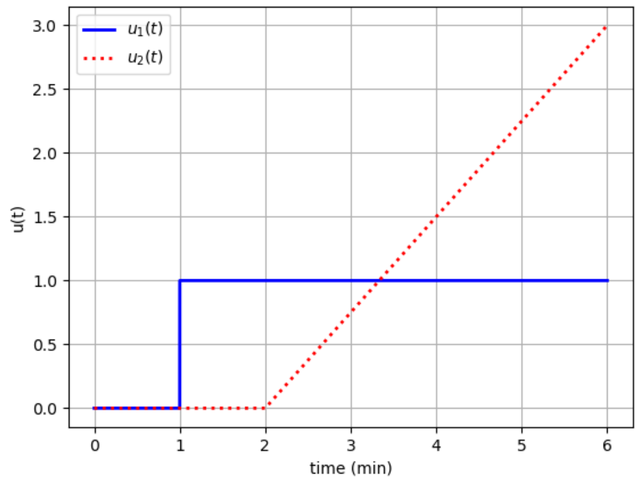
f(t) in Time Domain	*F(s)* in Laplace Domain
$\frac{df}{dt}$	$sF(s)-f(0)$
$\frac{d^n f}{dt^n}$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f^{(1)}(0)-\ldots-sf^{(n-2)}(0)-f^{(n-1)}(0)$
$\int f(t)$	$\frac{F(s)}{s}$
$f(t-t_0)\,S(t-t_0)$	$e^{-t_0 s}F(s)$

1. (30 pts) A process has two first-order transfer functions in parallel with inputs shown graphically below in the plot.



$$\frac{Y(s)}{U_1(s)} = G_1(s) = \frac{1}{s+2}$$

$$\frac{Y(s)}{U_2(s)} = G_2(s) = \frac{s\,e^{-s}}{2s+1}$$



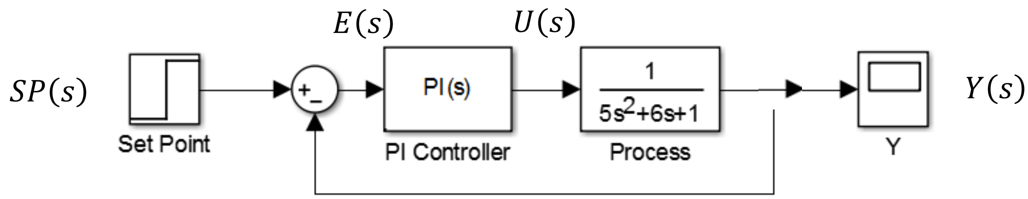
(a) What is the expression for $U_1(s)$ and $U_2(s)$?

(b) What is the expression for $y(t)$? Sketch the response $y(t)$ on the graph above.

Math hint: $e^a e^b = e^{a+b}$

2. (30 pts) A PI controller and second-order process transfer function create a closed loop response.

$$G(s) = \frac{1}{5s^2 + 6s + 1}$$



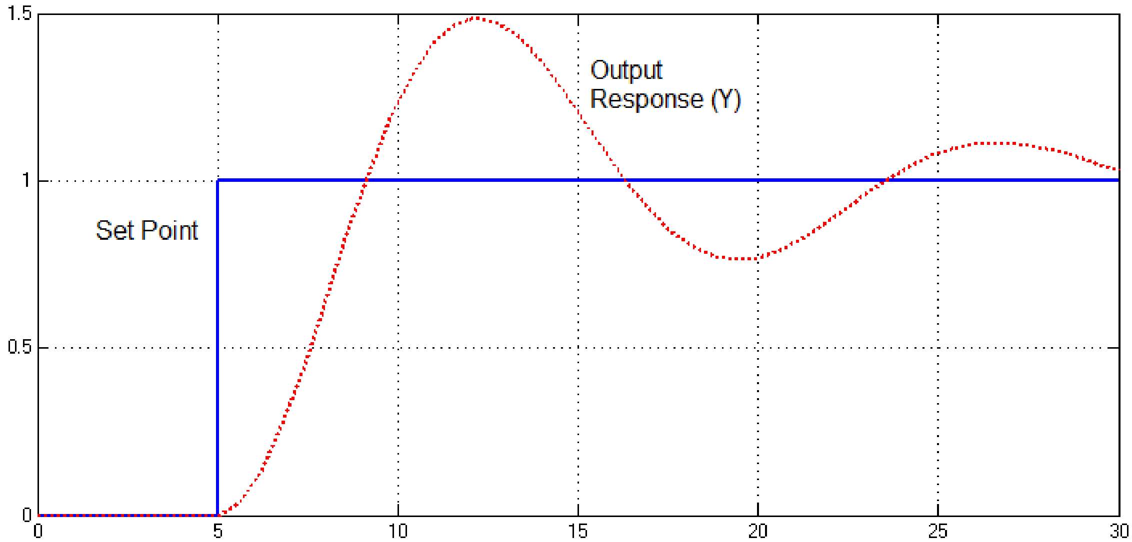
The objective is to find the value of K_c in the PI controller that will give a particular underdamped (oscillating) response. The value of the integral reset time is fixed at $\tau_I = 2$. The Laplace transform of a PI controller is shown below.

$$u(t) = u_{bias} + K_c e(t) + \int \frac{K_c}{\tau_I} e(t)$$

$$U(s) = K_c E(s) + \frac{K_c}{\tau_I s} E(s) = K_c \frac{\tau_I s + 1}{\tau_I s} E(s) \text{ with } PI(s) = \frac{U(s)}{E(s)}$$

The closed loop transfer function relates the Output Response (*Y(s)*) to the input change in Set Point (*SP(s)*). **What are the K_c values that give a stable controller?**

3. (40 pts) A PI controller has the following closed loop response.



a) What are the parameters of a second order system that approximate this closed loop response?

$$\frac{Y(s)}{SP(s)} = \frac{K}{\tau_s^2 s^2 + 2\zeta\tau_s s + 1} e^{-\theta s}$$

K:________________________________

τ_s:________________________________

θ:________________________________

ζ:________________________________

b) Write second order system as a state space model where u is the set point.

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

A:

B:

C:

D:
